F.Y.B.SC. PHYSICS

First Semester Paper I

Mechanics, Properties of Matter and Sound

Dr. Sanjay K. Tupe

Assistant Professor & Head department of Physics
Kalikadevi Arts, Comm & Science college, Shirur (K.),
Dist. – Beed, Maharashtra, Pin- 413249.

CHAPTER – 2

ELASTICITY

Introduction :

The force applied to a body can produce a change in shape or size or both in shape and size of the body, such a change is called as deformation. By applying suitable force, all bodies can be more or less deformed. Following are the simplest cases of deformation. i) Change in length, ii) Change in volume but no change in shape. iii) Change in shape but no change in volume. For changing the shape and size of the body their requires some force that force is called as deforming force. So that the deforming force is defined as the force which produces deformation in the body. **"Elasticity is the property possessed by a material body by virtue of which it regain its original condition when the applied deforming force are removed."** Bodies, which can recover competely their original condition on the removal of the deforming force are said to be perfectly elastic on the other hand bodies which do not show any tendency to recover their original condition are said to be plastic. Rubber cord, metal like steel, copper, silver fold etc. within the elastic limit are the examples of elastic body. Clay, wax, plasticene etc. are the examples of plastic body. **"The internal restoring force per unit area is called as stress."** S.I. Unit of stress is N/m^2.

C.G.S. Unit of stress is $dyne/cm^2$. Tensile stress (longitudinal stress), Volume stress, Shearing stress are the three types of stress. **"The change in the diamensions per unit original diamension is called strain."** strain is a ratio of two similar quantities. It is a pure number so strain does not have any units.Tensile strain (longitudinal strain), Volume strain, Shearing strain are the three types of stress.

According to hooks law: "Within the elastic limit, stress is directly proportional to the strain".

$\therefore$ We can write as. $\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$

S.I. unit of modulus of elasticity in N/m^2.

C.G.S. unit of modulus of elasticity is $dyne/cm^2$.

2.1 Moduli of Elasticity (Elastic constants) :

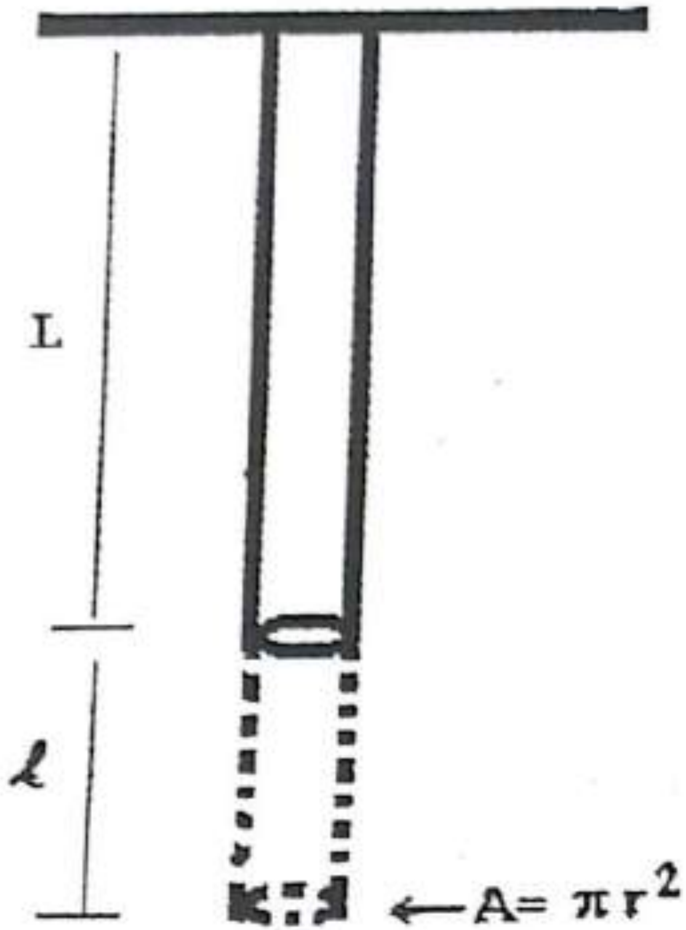
The modulus of elasticity is a material property, that describes its stiffness and is therefore one of the most important properties of solid materials. Mechanical deformation puts energy into a material. The energy is stored elastically or dissipated plastically. The way a material stores this energy is summarized in stress-strain curves. **Stress is defined as force per unit area and strain as elongation or contraction per unit length** When a material deforms elastically, the amount of deformation likewise depends on the size of the material, but the strain for a given stress is always the same and the two are related by Hooke´s Law (stress is directly proportional to strain) There are three types of modulus of elasticity for solid corressponding to three types of strain and stress

a) Young's Modulus (Y) :

"Within the elastic limit. It is the ratio of longitudinal stress to longitudinal strain"

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

Expression : Consider a metallic wire of original length L and radius r is suspended from rigid support. Let Mass. M is attached to free end of the wire such that its length is incresased by 'ℓ' as shown in Figure.



[Fig : Young's Modulus]

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\ell}{L}$$

$$\text{Longitudinal stress} = \frac{\text{Applied force}}{\text{Area of cross section}} = \frac{Mg}{\pi r^2}$$

Now by the defination,

$$Young's\ Modulus = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$Young's\ Modulus = \frac{\left(\frac{Mg}{\pi r^2}\right)}{\left(\frac{\ell}{L}\right)}$$

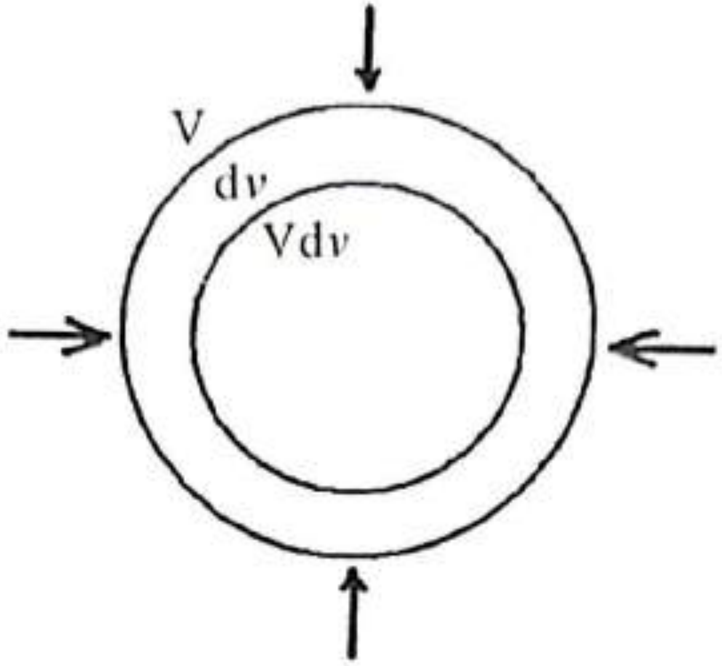
$$\boxed{\text{Young's Modulus} = \frac{MgL}{\pi r^2 \ell}}$$

b) Bulk Modulus (K) :

"Within the elastic limit, It is the ratio of Bulk stress to Bulk strain"

$$K = \frac{\text{Bulk stress}}{\text{Bulk strain}}$$

Consider a spherical body of original volume V is subjected to normal force F so that its changes by dv as shown in fig.



[Fig : Bulk Modulus]

Let A be surface area of body; then

$$\text{Bulk strain} = \frac{\text{Change in Volume}}{\text{Original Volume}} = \frac{dv}{V}$$

$$\text{Bulk stress} = \frac{\text{Applied force}}{\text{Area of cross section}} = \frac{F}{A} = dp, \textit{Change in pressure}$$

$$\text{Bulk } Modulus = \frac{\text{Bulk stress}}{\text{Bulk strain}}$$

$$\text{Bulk } Modulus\,(K) = \frac{\left(\frac{F}{A}\right)}{\left(\frac{dv}{\mathrm{V}}\right)}$$

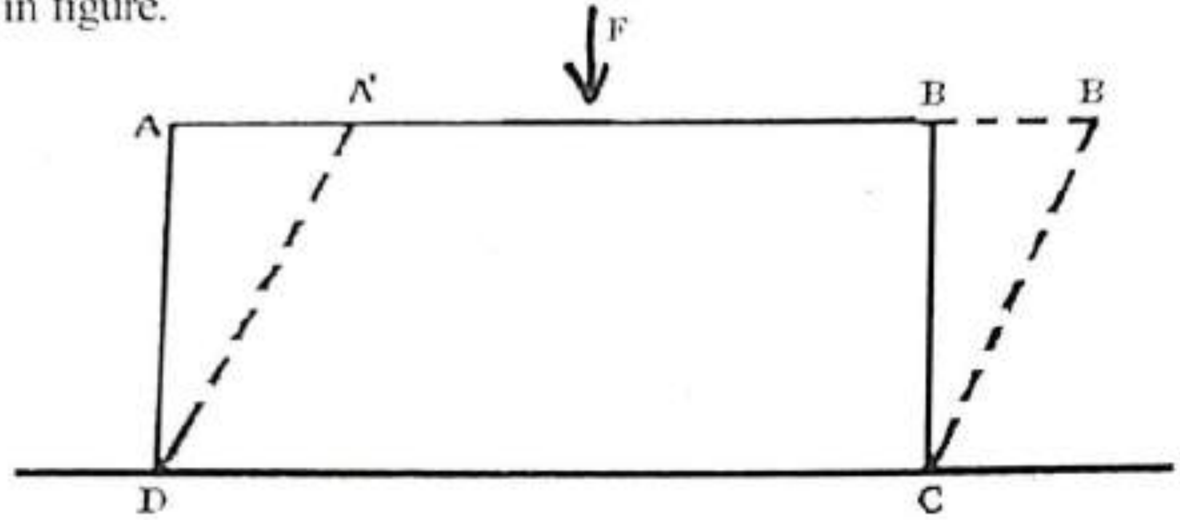
$$\boxed{\text{Bulk Modulus(K)} = \mathrm{V}\left(\frac{\mathrm{dp}}{\mathrm{dv}}\right)}$$

c) Modulus of rigidity (η):

"Within the elastic limit, It is the ratio of shearing stress to shearing strain"

$$Modulus\ of\ Rigidity = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

Consider a rectangular blocks ABCD with its lower and tangential force 'F' is applied to upper layer AB; So that upper layer AB is displaced laterally to $A'B'$ through small angle θ as shown in figure.



[Fig : Modulus of rigidity.]

$$\text{Shearing strain} = \frac{\text{Lateral displacement of layer}}{\text{Distance of layer from fixed layer}}$$

$$= \frac{AA'}{AD} = \tan\theta$$

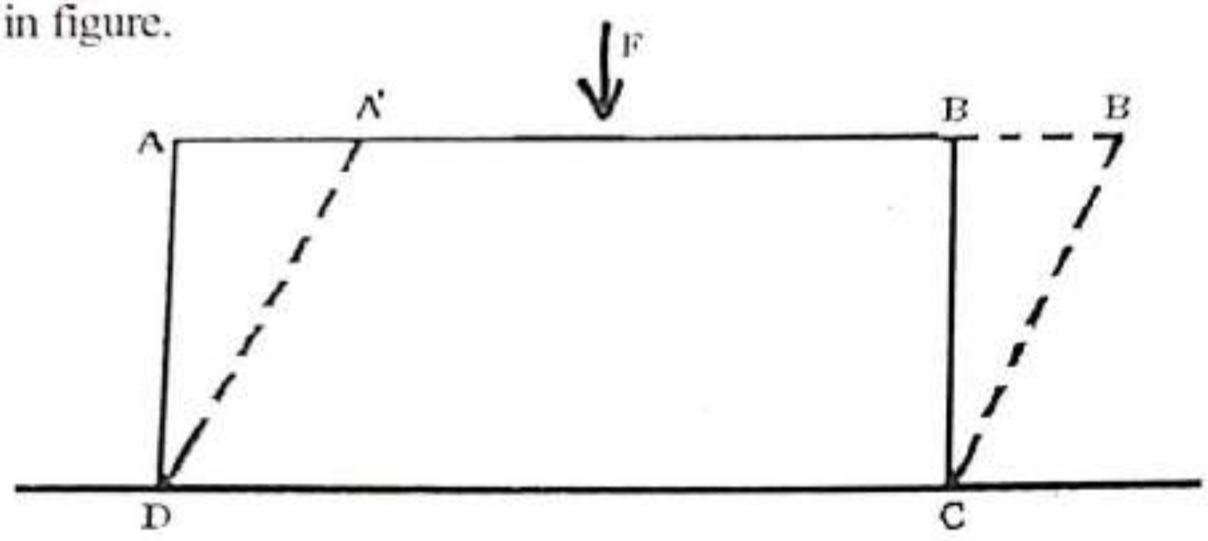
if θ is very small

c) **Modulus of rigidity** (η):

"Within the elastic limit, It is the ratio of shearing stress to shearing strain"

$$\textit{Modulus of Rigidity} = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

Consider a rectangular blocks ABCD with its lower and tangential force 'F' is applied to upper layer AB; So that upper layer AB is displaced laterally to $A'B'$ through small angle θ as shown in figure.



[Fig : Modulus of rigidity.]

$$\text{Shearing strain} = \frac{\text{Lateral displacement of layer}}{\text{Distance of layer from fixed layer}}$$

$$= \frac{AA'}{AD} = \tan\theta$$

if θ is very small

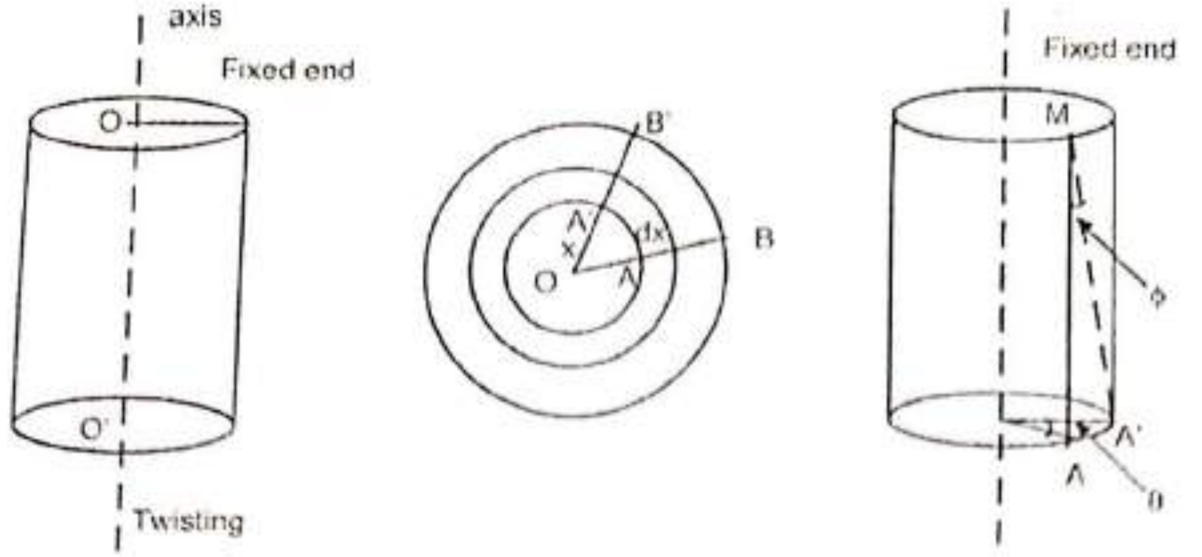


Fig: a **Fig: b** **Fig: c**

[Fig : Twisting couple on a cylinder]

The solid cylinder can be made up of large number of compactly hallow cylinder of increasing radius. When the cylinder is twisted through an angle θ a point B on outermost circumference displaces to B' while point A on an inner circumference of radius x and width dx displaced $A^{'}$

$BB^{'} \rangle AA^{'}$

To calculate net twisting couple acting on a solid cylinder

$\therefore$ Twisting couple acting on a solid cylinder will be,

$$\text{Twisting couple} = F.x \qquad \text{-------} \ (1)$$

$$\text{Shearing strain} = \frac{\text{lateral displacement of any layer}}{\text{it's distances from fixed layer}}$$

$$= \frac{AA'}{AM} = \tan\phi \quad \text{for } \phi \text{ is small}$$

$$\therefore \; = \phi \qquad \text{------- (2)}$$

$$\text{Shearing stress} = \frac{F}{A} \qquad \text{------- (3)}$$

The area of the shaded region is calculated

A = Area of the outer circle - Area of inner circle

$$A = \pi\,(x + dx)^2 - \pi x^2$$

$$A = \pi\,(x^2 + 2xdx + dx^2) - \pi x^2$$

$$A = \pi x^2 + 2x\pi\,dx + \pi\,dx^2 - \pi x^2$$

If dx is small then dx^2 is very small $\therefore\; dx^2 \approx 0$

$$A = 2x\pi\,dx$$

$$\therefore \text{Shearing stress} = \frac{F}{2x\pi.dx} \qquad \text{------- (4)}$$

$\therefore$ From equation (2) and (4) we can calculate

$$\text{Modulus rigidity} = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\eta = \frac{F}{\frac{2x\pi.dx}{\phi}} \qquad \text{------ (5)}$$

From fig (c) $\angle AOA' = \theta$ then

$$\theta = \frac{AA'}{x} \qquad AA' = \theta x \qquad \text{------ (6)}$$

$\angle AMA' = \theta$ then

$$\varphi = \frac{AA'}{l} \qquad AA' = \theta.\ell \qquad \text{------- (7)}$$

From (6) dn (7)

$$\phi.l = \theta.x \qquad \therefore \phi = \frac{\theta.x}{l}$$

Put this in equation (5) we get

$$\eta = \frac{F}{2\pi x.dx.\varphi}$$

$$= \frac{F}{2\pi x.dx.\frac{\theta x}{l}}$$

$$= \frac{Fl}{2\pi x^2.\theta.dx.}$$

$$\therefore F = \frac{2\pi x^2.\theta.dx}{l}.\eta \qquad \text{------- (8)}$$

substituting F in equation (1) we get,
Twisting couple = F. x

$$= 2\pi x^3 \frac{\theta}{l} dx.\ \eta$$

The total twisting couple can be

$$\text{Total twisting couple} = \int_0^r 2\pi x^3 \frac{\theta}{l} dx.\ \eta$$

$$= 2\pi \frac{\theta}{l} \eta \int_0^r x^3 dx$$

$$= 2\pi \frac{\theta}{l} \eta \frac{x^4}{x}$$

$$= \frac{\pi.\theta.\eta.x^4}{2l}$$

$$\frac{\text{Total twisting couple}}{\theta} = \frac{\pi.\eta.x^4}{2l}$$

L.H.S. of this equation gives twisting couple per unit twisting angle and is denoted by C

$$\boxed{C = \frac{\pi.\eta.x^4}{2l}}$$

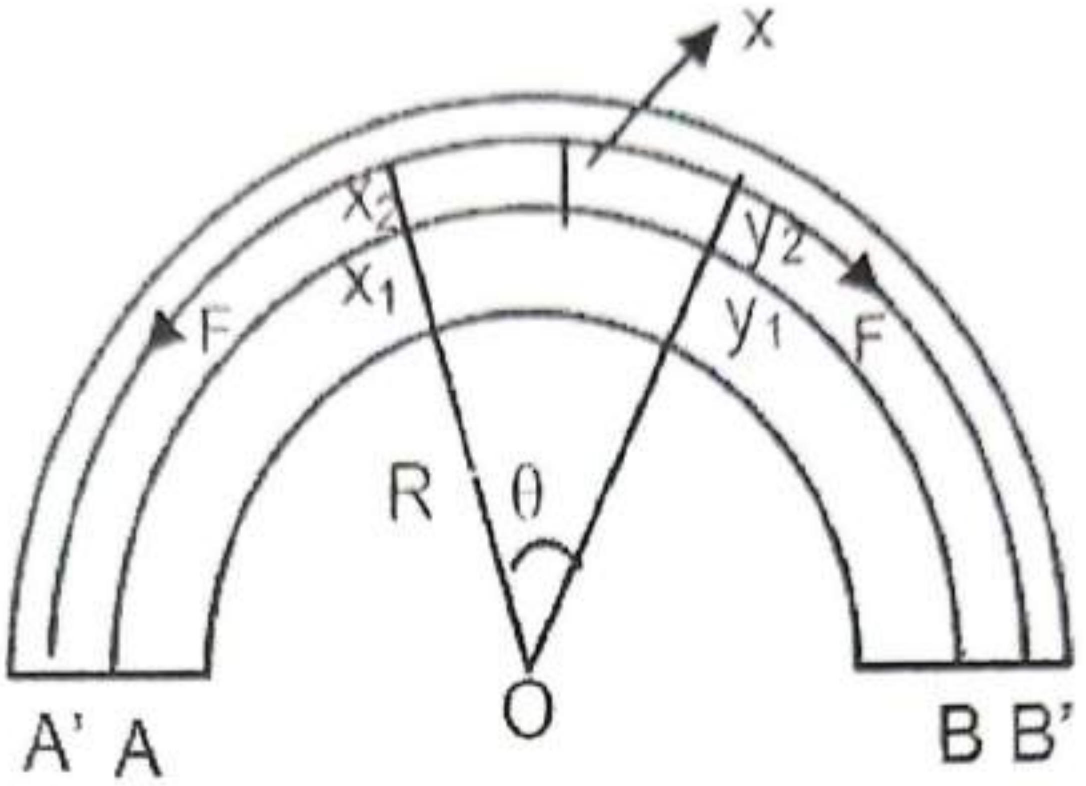
C is called torsional rigidity of the given solid cylinder.

2.3 Bending moment (M) of a beam :

A beam may be made of any material like wood, cement concrete, metals etc. Its cross section may be circular, rectangular or square. As compair to the surface area, length of the beam is very large. Such beam can support heavy loads i.e. R.C.C. construction of building.

"Beam is defined as a structure of uniform cross section with sufficiently large length".

Let us consider the beam is bent to form arc of circle of radius R. Whose center is O. AB is the neutral axis of the beam so the length of the beam remains unchanged. A'B' above AB which is at a distance x from AB is stretched.



[Fig : Bending moment of beam]

Let X_1Y_1 is a small segment of the neutral axis.

Thus

$OX_1 = OY_1 = R$

Sinces X_2Y_2 is longer than X_1Y_1

$\therefore X_2Y_2 - X_1Y_1$ = extension of X_1Y_1 on bending.

we have,

$$\frac{X_1Y_1}{R} = \theta = \frac{X_2Y_2}{(R+X)}$$

$\therefore X_2Y_2 = (R+X)\theta$

and $X_1Y_1 = R.\theta$

$\therefore$ Extension in X_1Y_1 on bending $= X_2Y_2 - X_1Y_1$

$$= (R+X)\theta - R.\theta$$

$$= R.\theta + X\theta - R.\theta$$

$$= X.\theta$$

$\therefore$ tensile or linear strain $= \dfrac{\text{Extension}}{\text{Initial length}}$

$$= \frac{X.\theta}{X_1Y_1}$$

$$= \frac{X.\theta}{R.\theta} \quad \text{------- (4)}$$

$$= \frac{X}{R}$$

tensile stress $= \frac{F}{A} = \frac{F}{\delta a}$

If young's modulus of matterial of the Y we get,

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}}$$

$$= \frac{F/\delta a}{x/R}$$

$$= \frac{F}{\delta a} \times \frac{R}{x}$$

$$\therefore F = Y.\delta a \times \frac{x}{R} \quad \text{------- (5)}$$

This is the restoring force acting on area da of filament A_1B_1.
Moment of this force about neutral axis

$$F.x = \left(Y.\delta a \times \frac{x}{R}\right).x$$

$$= \frac{Y.\delta a.x^2}{R} \qquad \text{------- (6)}$$

The sum of moment of all restoring force on all the filament about neutral axis, produced inside the bending beam. Its equillibrium position is called bending moment of the beam.

$\therefore$ Take the algebric sum of equation (6)

$$\therefore M = \sum \frac{Y.\delta a.x^2}{R}$$

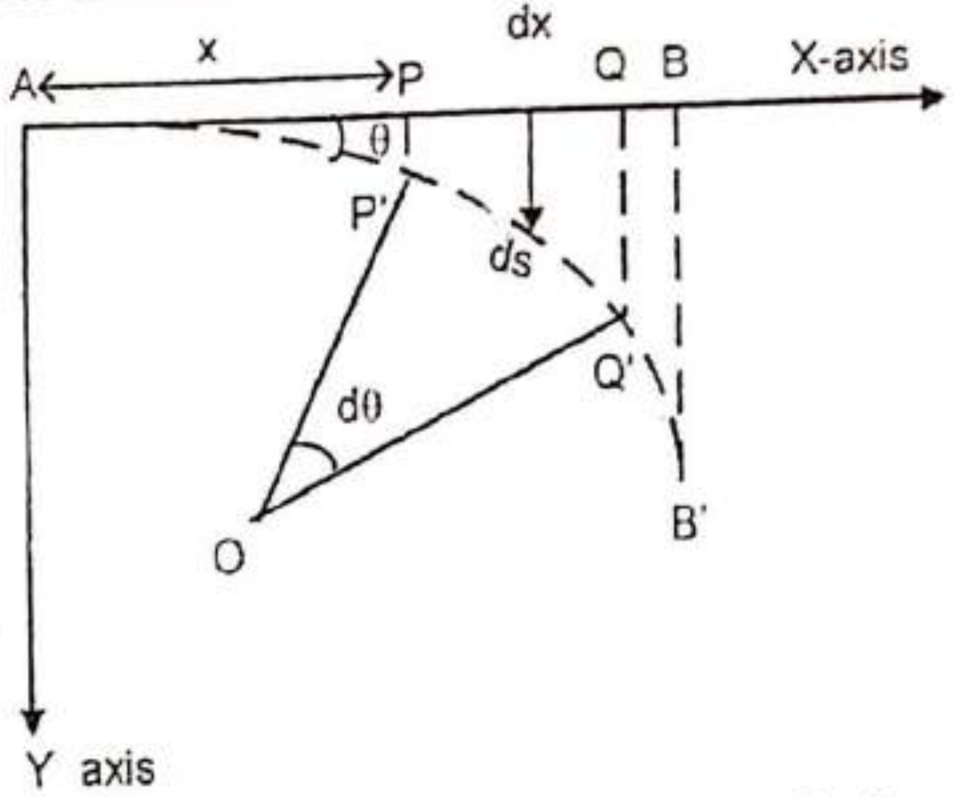
$$= \frac{Y}{R}\sum \delta a.x^2$$

The quantity $\sum \delta a.x^2$ is called geometrical moment of inertia (Ig) of the beam.

$$\boxed{M = \frac{Y}{R} \times Ig} \qquad \text{------- (7)}$$

This is the formula for bending moment of the beam.

2.4 Curvature of a beam or bar :



[Fig : Curvature of a beam or bar]

Let us consider a bar AB which is fixed at end A is bent along AB' to form a circular arc of the radius R, small section PQ of at distance x is depressed by y in downword direction we have

$$PQ = P'Q' = dx$$

for small depression

$$dq = ds = dx$$

$$d\theta = \frac{\text{arc length}}{\text{Radius}}$$

$$d\theta = \frac{ds}{R}$$

$$\frac{d\theta}{ds} = \frac{1}{R} \qquad \text{or,} \qquad \frac{d\theta}{dx} = \frac{1}{R} \qquad \text{------} \quad (2)$$

but for small angle q

$$\tan\theta = \theta = \frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{R}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{R}} \qquad \text{-------} \quad (3)$$

This is the formula for curvature of a beam of a bar.

2.5 Cantilever loaded at free end :

Def : "It is a beam fixed horizontally at one end and loaded at the other."

Let us consider a cantilever which consist of a bar of uniform cross section of length l one end of the bar is fixed to the table and other end is free.

Let a weight w = mg be suspended at its free end, then their are two cases

i) Assume that the cantilever to be weightless.

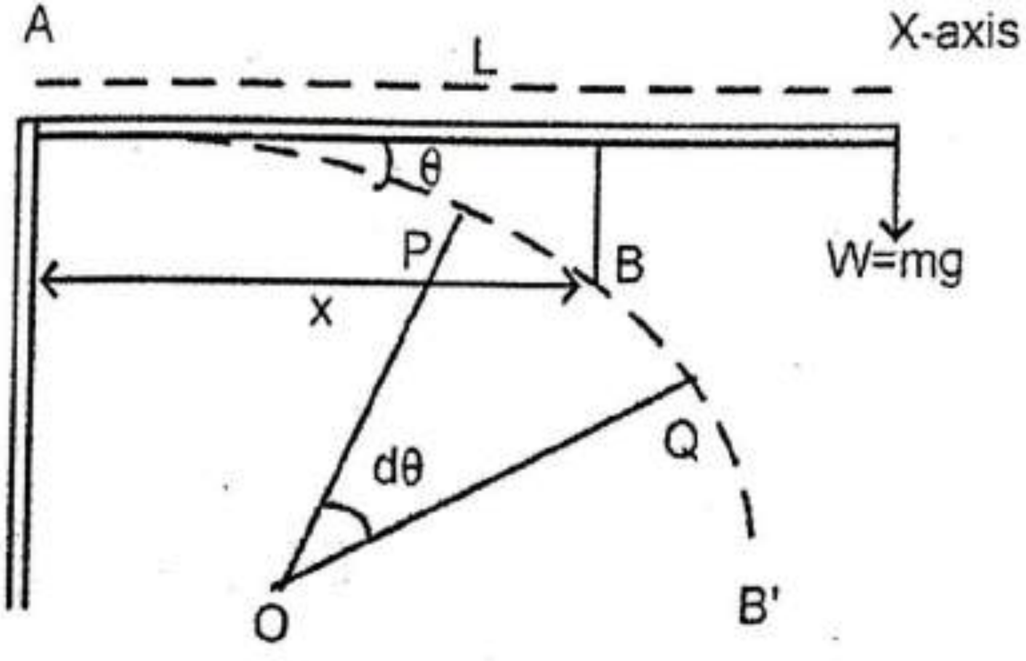
ii) Assume that the cantilever to be weighted.

In these two cases there requires to find out the youngs modulus.

(a) When weight of beam is ineffective :

Let PQ is be the small segment of the cantilever and R is its radius. Let x be the distances between center of PQ and fixed end A. AB' shows final position of neutral axis of the bar.

We have bending moment for PQ is,



[Fig : When weight of beam is ineffective]

$$M = \frac{Y.Ig}{R} \qquad \text{------- (1)}$$

where,

M - is the bending moment.

Y - is the young's modulus.

Ig - is the geometrical moment of inertia.

R - is the radius of the arc.

Since depression moment of force w for section PQ is

External depression moment = W(l-x)

In the equilllibrium state,

Bending moment of PQ = External depression moment

$$\frac{Y.Ig}{R} = W(l - x) \qquad \text{------- (2)}$$

but we know that, the formula for curvature of a beam or bar is

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

∴ equation (2) becomes,

$$\frac{d^2y}{dx^2} = \frac{W}{Y.Ig}(l - x) \qquad \text{------- (3)}$$

integrating equation (3) with respective x we get,

$$\int \frac{d^2y}{dx^2} dx = \int \frac{W}{Y.Ig}(l - x).dx$$

$$= \frac{W}{Y.Ig}\int (l - x).dx$$

$$= \frac{W}{Y.Ig}(lx - \frac{x^2}{2}) + K_1 \qquad \text{------- (4)}$$

To find constant of integration K_1 in equation (4) we know that at $x = 0$. i.e. at A depression is 0.

$$\therefore \theta = 0 \text{ and } \tan\theta = \frac{dy}{dx} = 0$$

$$\therefore K_1 = 0$$

$$\frac{dy}{dx} = \frac{W}{Y.Ig}\left(lx - \frac{x^2}{2}\right) \qquad \text{------- (5)}$$

To find y the value of depression at x from A, we again integrate equation (5) with respective x.

$$\therefore y = \frac{W}{Y.Ig}\int\left(lx - \frac{x^2}{2}\right)dx$$

$$= \frac{W}{Y.Ig}\left[\frac{lx^2}{2} - \frac{x^3}{3\times 2}\right] + K_2$$

$$= \frac{W}{Y.Ig}\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] + K_2 \qquad \text{------- (6)}$$

again sinces x = 0, y = 0, we get

$$K_2 = 0$$

$$\therefore y = \frac{W}{Y.Ig}\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] \qquad \text{-------} (7)$$

This is depressin at distances x from A. hences depression of bar B to B_1 can be obtained by substituting x = l.

$$y_B = \frac{W}{Y.Ig}\left[\frac{l^3}{2} - \frac{l^3}{6}\right]$$

$$= \frac{W}{Y.Ig}\left[\frac{3l^3 - l^3}{6}\right] = \frac{W}{Y.Ig}\left[\frac{2l^3}{6}\right]$$

$$y_B = \frac{W}{Y.Ig}\left[\frac{l^3}{3}\right] \qquad \text{-------} (8)$$

This is the formula for depression of a weightless cantilever when loaded by weight w at its free end.

If the cantilever is in the form of **rectangular bar of** a breadth b and thickness d then geometrical moment of inertia is given by,

$$I_g = \frac{bd^3}{12}$$

$$\therefore Y_B = \frac{W}{Y\left(\frac{bd^3}{12}\right)} \times \frac{l^3}{3} = \frac{12Wl^3}{3Ybd^3}$$

$$\therefore Y_B = \frac{4Wl^3}{Ybd^3}$$

$$\boxed{\therefore\ Y = \frac{4Wl^3}{Y_B \times b \times d^3}} \qquad \text{------- (9)}$$

This is the equation to calculate young's modulus of cantilever using bar of **rectangular** cross section.

If the cross section is **circular** then.

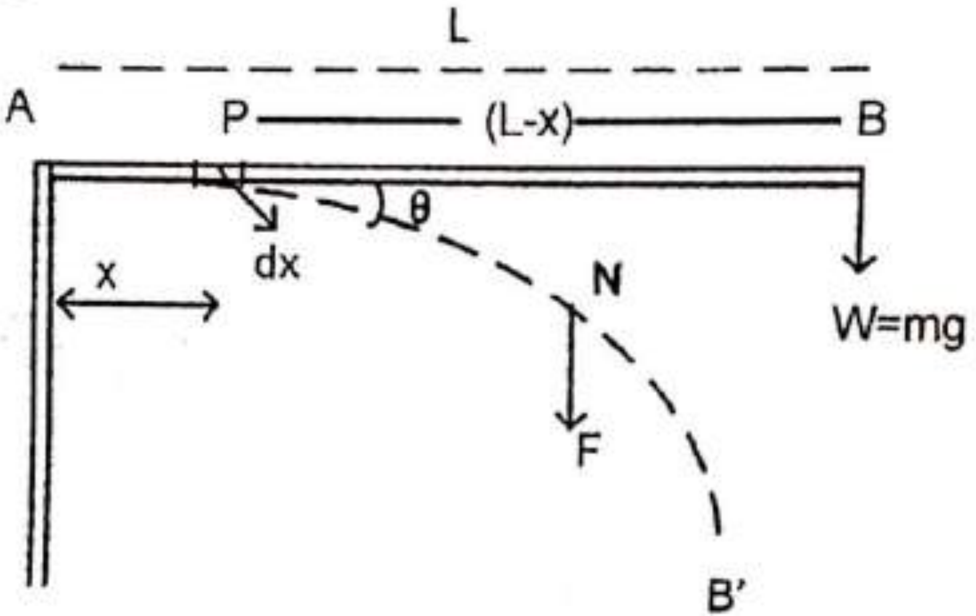
$$I_g = \frac{\pi r^4}{4}$$

$$\therefore Y_B = \frac{W}{Y \times \frac{\pi . r^4}{4}} \times \frac{l^3}{3}$$

$$= \frac{4}{3} \times \frac{Wl^3}{Y\pi . r^4}$$

$$\boxed{\therefore Y = \frac{4}{3} \times \frac{Wl^3}{Y\pi . r^4}} \qquad \text{-------} \ (10)$$

(b) When weight of beam is effective :



In the first case depression moment was only due to the applied weight W. In this case weight of the cantilever W_1 also contributes to the depression.

Consider a small section of length dx at P. It will be depressed by applying loading weight W at the free end B of the cantilever. In this case we also consider that the weight of the cantilever (W_1). Due to these weight of the cantilever their is additional force which gives the depression moment at P. These force F is due to weight of the section of cantilever of the length (l - x) and acting at the mid point N.

$$\text{Weight per unit length of cantilever} = \left(\frac{W_1}{l}\right)$$

$\therefore$ F= weight of section of length (l-x)

F= weight per unit length x sectional length

$$F = \left(\frac{W_1}{l}\right) \times (l - x) \qquad \text{------- (1)}$$

Sinces, these force F is acting at the midpoint (N)

$\therefore$ The length of the bar between P and N point is = $(\frac{l-x}{2})$

The moment of the force F about point N = F(l-x)

The moment of force F at out point P will be given $= \frac{F(l-x)}{2}$

using equation (1) we can write as $= \left(\frac{W_1}{l}\right)(l-x)\frac{(l-x)}{2}$

$$= \frac{W_1(l-x)^2}{2l} \qquad \text{------- (2)}$$

The moment of weight W about P will be given

$$= W(l\text{-}x) \qquad \text{------- (3)}$$

Thus we have to find the total depression moment about P.

$\therefore$ The total depression moment about P

= Moment of force F + Moment of weight

$$= \frac{W_1(l-x)^2}{2l} + W(l-x) \qquad \text{------- (4)}$$

but we know that,

The bending moment (M) of the bar is given by,

$$M = \frac{Y}{R} Ig \qquad \text{------- (5)}$$

where,

Y - is the youngs modulus

R - is the radius of curvature at P.

Ig - Geometerical moment of inertia.

but, the total depression moment about P is nothing but the bending moment (M) of the bar.

∴ From equation (4) and (5) we can write as,

The total depression moment about P = bending moment of bar.

$$\therefore \frac{Y}{R} Ig = \frac{W_1 (l - x)^2}{2l} + W(l - x)$$

but we know that the formula for curvature of beam or bar

$$\frac{1}{R} = \frac{d^2 y}{dx^2}$$

put the value of 1/R in above equation we get,

$$Y.Ig\frac{d^2y}{dx^2}=\frac{W_1(l-x)^2}{2l}+W(l-x)$$

$$\frac{d^2y}{dx^2}=\frac{W_1(l-x)^2}{2l.Y.Ig}+\frac{W(l-x)}{Y.Ig} \qquad \text{------- (6)}$$

Integrating above equation with respective x we get

$$\frac{dy}{dx}=\int\frac{W_1(l-x)^2}{2l.Y.Ig}dx+\int\frac{W(l-x)}{Y.Ig}dx$$

$$=\int\frac{W_1}{2l.Y.Ig}\left(l^2-2lx+x^2\right)dx+\int\frac{W}{Y.Ig}(l-x)dx$$

$$=\frac{W_1}{2l.Y.Ig}\left(l^2x-\frac{2lx^2}{2}+\frac{x^3}{3}\right)+\frac{W}{Y.Ig}(lx-\frac{x^2}{2})+k_1 \qquad \text{--------(7)}$$

Sinces $\frac{dy}{dx}=0$ at x = 0

$\therefore$ we get k_1 =0

$$\therefore\frac{dy}{dx}=\frac{W_1}{2l.Y.Ig}\left(l^2x-lx^2+\frac{x^3}{3}\right)+\frac{W}{Y.Ig}(lx-\frac{x^2}{2}) \qquad \text{------- (8)}$$

again integrating above equation with respective x we get,

$$\int \frac{dy}{dx} dx = \int \frac{W_1}{2l.Y.Ig}\left(l^2x - lx^2 + \frac{x^3}{3}\right)dx + \frac{W}{Y.Ig}\int (lx - \frac{x^2}{2})dx$$

$$\therefore Y = \frac{W_1}{2Y.Ig l}\int\left(l^2x - lx^2 + \frac{x^3}{3}\right)dx + \frac{W}{Y.Ig}\int (lx - \frac{x^2}{2})dx$$

$$Y = \frac{W_1}{2Y.Igl}\left[\frac{l^2x^2}{2} - \frac{lx^3}{3} + \frac{x^4}{12}\right] + \frac{W}{Y.Ig}\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] + K_2$$

where K_2 is the integration constant

Sinces,

$y = 0$, at $x = 0 \therefore K_2 = 0$

$$\therefore y = \frac{W_1}{2Y.Igl}\left[\frac{l^2x^2}{2} - \frac{lx^3}{3} + \frac{x^4}{12}\right] + \frac{W}{Y.Ig}\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] \quad \text{------- (9)}$$

The net depression y_B at end B is calculated by substituting x = l in equation (9) we get

$$\therefore y = \frac{W_1}{2Y.Igl}\left[\frac{l^4}{2} - \frac{l^4}{3} + \frac{l^4}{12}\right] + \frac{W}{Y.Ig}\left[\frac{l^3}{2} - \frac{l^3}{6}\right]$$

$$= \frac{W_1}{2Y.Igl}\left[\frac{6l^4 - 4l^4 - l^4}{12}\right] + \frac{W}{Y.Ig}\left[\frac{3l^3 - l^3}{6}\right]$$

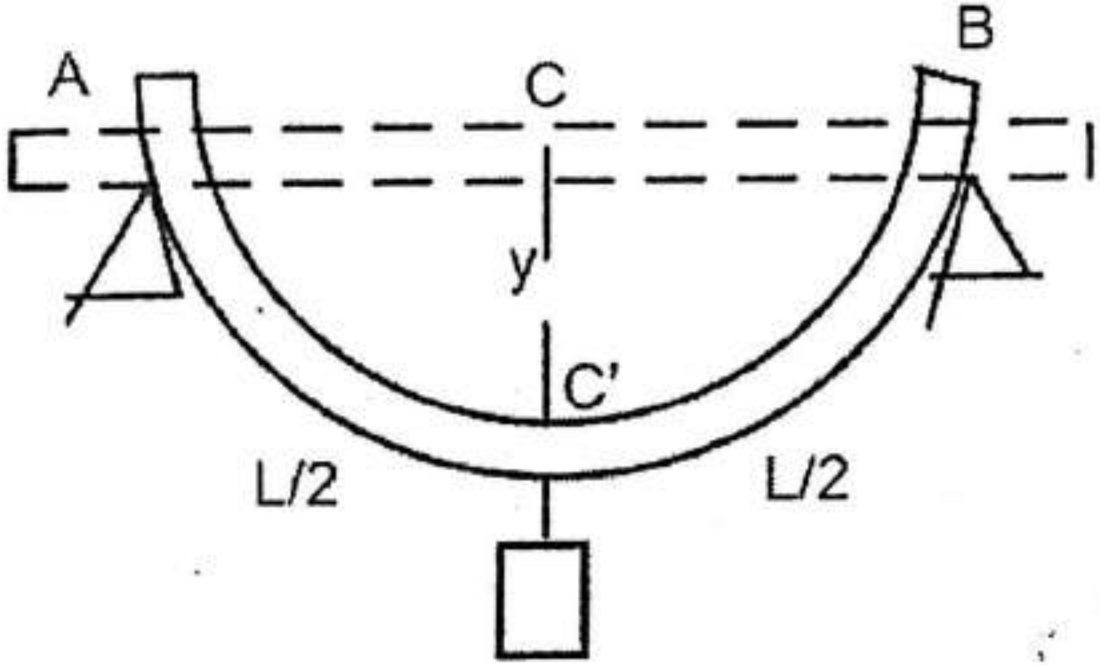
$$= \frac{W_1}{2Y.Igl}\left[\frac{3l^4}{12}\right] + \frac{W}{Y.Ig}\left[\frac{2l^3}{6}\right]$$

$$= \frac{W_1}{2Y.Ig}\left[\frac{l^4}{4}\right] + \frac{W}{Y.Ig}\left[\frac{l^3}{3}\right]$$

$$\boxed{y = \frac{l^3}{3Y.Ig}\left[W + \frac{3W_1}{8}\right]} \qquad \text{------- (10)}$$

2.6 Beam is supported to both the ends :

Let us consider a bar of uniform cross section be supported horizontally on two knife edges at A and B. Let us consider that the length of the bar is l and C is the center of the rod which is also center of AB. Let a load W be attached to the rod at its center so that $CC' = y =$ maximum depression of the rod.



[Fig : Beam supported to both the ends]

The weight W be balanced by W/2 at point A and W/2 at B then AC' or BC' shows the depression of bar.

We know that the depression of a weightless cantilever, loaded at its free end and is given by,

$$\text{depression} = \frac{(\text{load})}{YIg} \times \frac{(\text{length})^3}{3} \qquad \text{------- (1)}$$

$$= \frac{W}{YIg} \times \frac{l^3}{3}$$

but in this case load is W/2 and length is $l/2$

$$\therefore y = \frac{W/2}{YIg} \times \frac{l^3/8}{3}$$

$$= \frac{Wl^3}{48YIg} \qquad \text{------- (2)}$$

where,

Y - is the young's modulus of material of a bar

y - is the depression of a bar.

Ig - is the geometerical moment of inertia.

W - is the applied weight on the bar.

l - is the length of the bar from the two knief edges.

If the rod is rectangular in cross section of breadth b and depth d then geometrical moment of inertia is given by

$$Ig = \frac{bd^3}{12} \qquad \text{------- (3)}$$

put this equation in equation (2) we get

$$y = \frac{Wl^3}{48\,Y\,(\frac{bd^3}{12})} \qquad \text{------- (4)}$$

$$\therefore y = \frac{Wl^3}{4\,Y\,bd^3} \qquad \text{------- (5)}$$

If the rod is circular in cross section then the geometerical moment of inertia is given by

$$Ig = \frac{\pi.r^4}{4} \qquad \text{------- (6)}$$

substituting this equation in equation (2) we get,

$$\boxed{y = \frac{Wl^3}{48\,Y\,(\frac{\pi.r^4}{4})}}$$

$$y = \frac{Wl^3}{12\,Y\,\pi.r^4}$$

$$\therefore y = \frac{Wl^3}{12\,Y\,\pi.r^4} \quad \text{------- (7)}$$

This formula is used to calculate y, if material is in the form of a rod of circular cross section.

:: Solved Problems ::

Ex.1. A cylinderical rod of diameter 14mm rest on two knief-edges 0.8m apart and a load of 1kg is suspended from its mid-point. Neglecting the weight of the rod, calculate the depression of the mid-point. If Y for its material be 2.04×10^{11} N/m².

Given :

$l = 0.8$ m, $r = 14 \times 10^{-3}$ m/2 = 0.014/2 = 0.007m.

$W = 1$ kg = 9.81N, $Y = 2.04 \times 10^{11}$ N/m²

To Calculate : y = ?

Solution :

Sinces, the depression of the mid-point of the bar is given by

$$y = \frac{Wl^3}{12\,Y\,\pi.r^4}$$

This formula is in the form of a rod of circular cross section

$$y = \frac{9.81 \times (0.8)^3}{12 \times 2.04 \times 10^{11} \times 3.14 (0.007)^4}$$

$$= \frac{5.02272}{12 \times 2.04 \times 10^{11} \times 3.14 \times 2.40 \times 10^{-2}}$$

$$= \frac{5.02272}{184.55815 \times 10^{-2}}$$

$= 0.0272 \times 10^{-3}$m = 0.0272mm

Ex.7. A steel rod of circular cross section of radius 1cm is rigidly fixed at one end and a load of 8 kg is at the other end which is 100cm from the fixed end. Calculate depression of end. (Given : Y = 20×10^{11} dynes/cm²)

Given :

r = 1 cm,

l = 100cm

W = 8000 x 980 dyne

Y = = 20×10^{11} dynes/cm²

To calculate : $Y_B = ?$

$$y = \frac{4Wl^3}{3Y\pi.r^4}$$

$$y_B = \frac{4Wl^3}{3Y\pi.r^4}$$

$$= \frac{4 \times 8000 \times 980 \times (100)^3}{3 \times 20 \times 10^{11} \times 3.14 \times (1)^4}$$

= 1.66 cm

:: Multiple Choice Questions ::

1) If the work done in stretching a wire by 1 mm is 2 J, the work necessary for stretching another wire of the same material but with double the radius of cross-section and half the length by 1 mm is in joules

   (a) 16 (b) 8
   (c) 4 (d) 1/4

2) The modulus of elasticity is dimensionally equivalent to

   (a) Strain **(b) Stress**
   (c) Surface tension (d) Poisson's ratio

3) If by applying a force, the shape of a body is changed, then the corresponding stress is known as

   (a) Tensile stress (b) Bulk stress
   (c) Shearing stress (d) Compressive stress

4) The bulk modulus of a gas is 6×10^3 N/m² the additional pressure needed to reduce the volume of the gas by 10% is

(a) 300 N/m² (b) 400 N/m²

(c) 1000 N/m² **(d) 600 N/m²**

5) According to Hooke's law of elasticity, within elastic limits, if the stress is increased, the ratio of stress to strain.

(a) Increases (b) Decreases

(c) Becomes zero **(d) Remains constant**

6) One end of a steel wire of area of cross-section 3 mm² is attached to the ceiling of an elevator moving up with an acceleration of 2.2 m/s² if a load of 8 kg is attached at its free end, then the stress developed in the wire will be

(a) 8×10^6 N/m² (b) 16×10^6 N/m²

(c) 20×10^6 N/m² **(d) 32×10^6 N/m²**

7) The following four wires of length L and the radius r are made of same material. Which of these will have the largest extension when the same tension is applied.

(a) L = 50 cm, r = 0.25 mm

(b) L = 100 cm, r = 0.5 mm

(c) L = 200 cm, r = 1 mm

(d) L = 3000 cm, r = 1.5 mm

8) A wire of length 'L' and cross-sectional area A is mad of a material of Young's modulus Y. if the wire is stretched by an amount x then work done is

(a) F.x (b) $\frac{1}{2}(F/L)$

(c) $\frac{1}{2}\frac{YA}{L}x^2$ (d) $\frac{YA}{L}x$

9) The symbols, Y, K and h represent the Young's modulus, bulk modulus and rigidity modulus of the material of a body. If $\eta = 3K$, then

(a) Y = 2.5K (b) Y = 3.5K

(c) Y = 4.5K (d) Y = 9K/5

10) Two wires have the same material and length, but their masses are in the ration of 4:3. If they are stretched by the same force, their elongations will be in the ratio of

(a) 2:3 **(b) 3:4**

(c) 4:3 (d) 9:16

11) Two wires A and B are of the same length. The diameters are in the ratio 1:2 and the Young's modulus are in ratio 2:1. If they are pulled by the same force, then their elongations will be in ratio.

(a) 4:1
(b) 1:4
(c) 1:2
(d) 2:1

12) Shearing strain is given by

(a) Deforming force
(b) Shape of shear
(c) Angle of shear
(d) Change in volume of the body

13) If a wire having initial diameter of 2 mm produced the longitudinal strain of 0.1%, then the final diameter of wire will be ($\sigma = 0.5$)

(a) 2.002 mm
(b) 1.998 mm
(c) 1.999 mm
(d) 2.001 mm

Dear students read & write minimum three slides daily carefully.
If any doubt contact me or when we meet we will discuss.

Thank You