T.Y.B.SC. PHYSICS

Fifth Semester Paper XV

Classical & Quantum Mechanics

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CHAPTER - 3

Wave Particle Duality

3.1 Introduction:

Classical mechanics failed to explain phenomena like atomic spectra, blackbody radiation, photoelectric effect, Compton effect and specific heat capacity of solids. The inadequacy of classical mechanics led to the development of Quantum Mechanics.

(i) The Hydrogen Atom and the Bohr Model

As the first example of the failure of classical physics to account for observed phenomena, we consider the case of the hydrogen atom. Rutherford model failed to explain two main observational features of the hydrogen atom:

(a) Stability of hydrogen atom :

An electron in a curved orbit is accelerated and hence must radiate. As it radiates its energy away, the radius of its orbit must decrease until eventually it collapses into the nucleus. Thus, the atom cannot be stable. But most of the atoms are stable.

(b) the spectrum of its radiation in hydrogen atom :

The second discrepancy involves the observed radiation spectrum. The frequency of the radiated energy should be the same as the orbiting frequency. As the electron orbit collapses, its orbiting frequency increases continuously. We might thus expect the spectrum of radiation emitted by excited hydrogen atoms to be continuous. In contrast, the experimentally observed spectrum consists of families of discrete lines.

Bohr provided an explanation for both the spectral discreteness and the observed stability. He proposed that in solving for the orbital motion of the electron in its hydrogenic orbit one should impose an added condition:

The angular momentum of the electron must be equal to some integer multiple of it.

The quantisation implies that the laws of classical mechanics and of classical electromagnetism are inapplicable at the atomic level.

(ii) Black Body Radiation :

The observed variation of the spectral intensity I (v) (power per unit area per unit frequency) of blackbody radiation as a function of frequency v. From the curves we note that

(i) The intensity reaches a maximum at some frequency v_m.

(ii) The frequency v_m, as well as the height of the peak, increase with temperature.

The application of statistical thermodynamics and the ordinary laws of mechanics and electromagnetic theory led to the Rayleigh-Jeans formula. This law, except for very low frequencies, is in total disagreement with experimental results. The law predicts an infinite amount of radiated intensity. Actually, the total radiated intensity is finite. Max Planck resolved this controversy by postulating that the exchange of energy between atoms and radiation involves discrete amounts of energy.

3.2 De-Broglie's hypothesis for matter waves :

Louis De-Broglie suggested that :

"Matter considered to be made up of discrete particles such as atoms and molecules may also behave like wave under propoer conditions." He called them matter waves.

The wave nature of radiation has been well established by the phenomenan of diffraction and interference. Electrons, light rays or x-rays have also been shown to exhibit the diffraction phenomenon. This leads to the view that electrons like the light may also have wave properties associated with them. It means that an electron has dual nature, particle and wave.

Consider a moving particle, it has wave properties associated with it. Then the wavelength 'λ' associated with momentum 'P' then we have.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{----- (1)}$$

Here, m is mass of particle moving with velocity v & h is plancks constant.

Now we know the relativistic mass,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The waves thus associated with moving particles are known as matter waves.

3.3 De-Broglie's wavelength in terms of energy and temperature :

By using plancks theory of radiation the energy of photon is given by

$$E = h\upsilon = \frac{hc}{\lambda} \qquad \text{------ (2)}$$

Where υ is the frequency, C is the velocity of light in vaccum and λ is wavelength.

$$\therefore \lambda = \frac{hc}{E} \qquad \text{----- (3)}$$

If a particle of mass m is converted into energy, the equivalent energy is given by Einsteins mass-energy relation as,

$$E = mc^2$$

Therefore, eq^n (3) may be written as

$$\lambda = \frac{hc}{mc^2} = \frac{h}{mc} \qquad \text{----- (4)}$$

But mc = p, the momentum associated with quantum

$$\therefore \lambda = \frac{h}{p} \qquad \text{----- (5)}$$

De Broglie postulated that what is true for an energy packet which we call as quantum or photon is also true for the material particle. Thus, for a particle of mass m moving with velocity v. we have, $p = mv$

then, $$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

The kinetic energy E_K is given by

$$E_K = \frac{1}{2} mv^2$$ multiply & divide by m on R.H.S. we get,

$$E_K = \frac{1}{2}\frac{m^2 v^2}{m}$$

$$E_K = \frac{p^2}{2m}$$

$$p^2 = E_K 2m$$

or $$p^2 = 2m E_K$$

$$\therefore P = \sqrt{2m E_K}$$

or equation (6) can be written as,

$$\lambda = \frac{h}{\sqrt{2mE_K}} \qquad \text{----- (7)}$$

If material particle are in thermal equilibrium at associated temperature T then,

$$E_K = \frac{3}{2}KT$$

where, K is Boltzmann's const = 1.38×10^{-23} J/k

Then eqn (7) becomes,

$$\lambda = \frac{h}{\sqrt{2m\frac{3}{2}KT}}$$

or

$$\lambda = \frac{h}{\sqrt{3mKT}} \qquad \text{------ (8)}$$

3.4 De-Broglie phase velocity and particle velocity (relation between them):

A wave is a disturbance from equilibrium condition that travels or propagates with time from the region of space to another. The original displacement gives rise to an elastic force in the material adjecent to it, then the particle is displaced and then the next and so on Therefore, every particle begins its vibration a little later than its predecessor. Thus, there is a progressive change of phase from one particle to the next. The phase relationship of these particles is called as wave and the velocity with which planes of constant phase propagates through the medium is known as wave velocity or phase velocity.

Therefore, "The velocity of advancement of monochromatic wave through a medium is called wave velocity."

Let us consider the equation of plane progressive wave is as,

$$y = a\sin(\omega t - kx)$$

where $\omega = \frac{2\pi}{T} = 2\pi\upsilon$ is the angular frequency & $K = \frac{2\pi}{\lambda}$ is the propagation constant.

The term (ωt - kx) represents the phase of the wave motion. Hence, the plances of constant phase are written as,

$$\omega t - kx = \text{constant}$$

Differentiating w.r.t. time we get,

$$\omega t - k\frac{dx}{dt} = 0$$

or

$$\frac{dx}{dt} = \frac{\omega}{k} = u \qquad \text{----- (1)}$$

where $u = \frac{dx}{dt}$ is called as phase velocity or wave velocity.

Thus, it is the ratio of angular frequency w to the propagation constant K & u is the velocity with which a plane progressive wavefront travels forward.

The phase velocity, $u = \upsilon\lambda$ ----- (2)

where υ & λ are frequency and wavelength of the wave,

If E is the energy of the wave, then its frequency υ is given by,

$$E = h\upsilon \qquad \text{or} \qquad \upsilon = \frac{E}{h} \qquad \text{----- (3)}$$

DeBraglie wavelength of the material particle is given by,

$$\lambda = \frac{h}{mv} \qquad \text{----- (4)}$$

Therefore, from eqn (2), the phase velocity of the associated DeBroglie wave,

$$u = \upsilon\lambda = \frac{E}{h}.\frac{h}{mv}$$

or

$$u = \frac{E}{mv} \qquad \text{----- (5)}$$

But from Einstein's mass energy relation $E = mc^2$,

$$u = \upsilon\lambda = \frac{E}{h} \cdot \frac{h}{mv}$$

or $$u = \frac{E}{mv} \qquad \text{----- (5)}$$

But from Einstein's mass energy relation $E = mc^2$,

$$\therefore\ u = \frac{E}{mv} = \frac{mc^2}{mv} = \frac{c^2}{v} \qquad \text{----- (6)}$$

Here v is velocity of particle,

Since c>> υ.

Equation (6) shows the phase velocity of associated wave is greater than c, the velocity of light. It indicates that the associated wave with the particle travels faster than the particle itself. Thus, the particle will be left far behind. Obviously, a monochromatic DeBroglie wave cannot transport a particle or carry energy.

3.5 Group velocity :

The phase velocity of a wave associated with a particle comes out to be greater than the velocity of light. This difficulty can be solved by assuming each moving particle of matter to consist of a group of wave or wave packet, rather than a single wave train.

A wavegroup corresponding to a certain wave length λ consist of a number of component waves of slightly different wavelengths in the neighbourhood of λ superimposed upon each other. The mutual interference between component waves results in the variation of amplitude that defines the shape of the wave packet. The component waves interfere constructively over only a small region of space, outside of which they interfere destructively and hence the amplitude reduce to zero rapidly. Thus, the resultant wave pattern consist of points of maximum amplitude and points of minimum amplitude. Between any two constructive minima, there is a position of maximum amplitude. The dotted loop represents a group of waves or a typical wave pocket, shows in fig. This group of waves moves forward in the medium with a velocity called the group velocity.

Thus, **Group velocity** is the velocity with which the slowly varrying envelope of the modulated pattern due to a group of waves travel in a medium."

The importance of the group velocity lies in the fact that it is the velocity with which the energy in the wave group is transmitted. Consider a group of waves which consist of only two components of equal amplitude 'a' but slightly different angular frequencies ω_1 & ω_2 and propagation constants k_1 & k_2. Then we have,

$$y_1 = a \sin(\omega_1 t - k_1 x)$$

$$y_2 = a \sin(\omega_2 t - k_2 x)$$

The resultant amplitude due to superposition is given by,

$$y = y_1 + y_2$$

$$y = a \sin(\omega_1 t - k_1 x) + a \sin(\omega_2 t - k_2 x)$$

$$y = a [\sin(\omega_1 t - k_1 x) + \sin(\omega_2 x - k_2 x)]$$

$$y = a\left[2\sin\left\{\frac{\omega_1 t - k_1 x + \omega_2 t - k_2 x}{2}\right\}\right]$$

$$\cos\left\{\frac{\omega_1 t - k_1 x - \omega_2 t + k_2 x}{2}\right\}$$

$$y = 2a\sin\left\{\frac{(\omega_1 + \omega_2)}{2}t - \frac{(k_1 + k_2)x}{2}\right\}$$

$$\cos\left\{\frac{(\omega_1 + \omega_2)}{2}t - \frac{(k_1 + k_2)x}{2}\right\}$$

or $$y = 2a\cos\left\{\frac{(\omega_1 - \omega_2)}{2}t - \frac{(k_1 - k_2)x}{2}\right\}$$

$$\sin\left\{\frac{(\omega_1 + \omega_2)}{2}t - \frac{(k_1 + k_2)^x}{2}\right\} \qquad \text{----- (7)}$$

This equation represents a wave system of amplitude

$$A = 2a\cos\left\{\frac{(\omega_1-\omega_2)}{2}t-\frac{(k_1-k_2)}{2}x\right\}$$

which is modulated both in space & time.

Now equation (7) can be written as,

$$y = 2a\sin(\omega t - kx)\cos\left(\frac{\Delta w}{2}t-\frac{\Delta k}{2}x\right) \qquad \text{-----}\ (8)$$

$$\omega = \frac{\omega_1+\omega_2}{2},\ k = \frac{k_1+k_2}{2}\ \&\ \Delta\omega = \omega_1-\omega_2,\quad \Delta k = k_1-k_2$$

Thus, 1) A wave of frequency w, propagation constant k & velocity.

$$u = \frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \nu\lambda \qquad \text{-----}\ (9)$$

which is the phase velocity or wave velocity :

2) **Another wave of frequency $\frac{\Delta\omega}{2}$, Propagation constant $\frac{\Delta k}{2}$ and velocity**

$$V_g = \frac{\Delta\omega}{\Delta K}, \textbf{ then } \frac{\Delta\omega}{2} = \frac{(\omega_1 - \omega_2)}{2}$$

$$\& \ \frac{\Delta K}{2} = \frac{(K_1 - K_2)}{2}$$

Therefore,
$$V_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\Delta\omega}{\Delta k} \ or \ \frac{\partial\omega}{\partial k} \qquad \text{---- (10)}$$

Equation (10) is called as group velocity.

If a group contains a number of frequency components in a very small frequency interval, then,

$$V_g = \frac{\partial\omega}{\partial k}$$

$$\frac{\partial(2\pi\nu)}{\partial(2\pi/\lambda)} = \frac{2\pi\,\partial\nu}{2\pi\,\partial\left(\frac{1}{\lambda}\right)}$$

$$\therefore V_g = -\lambda^2 \frac{\partial\nu}{\partial\lambda} \qquad \text{---- (11)}$$

or $$G = V_g = -\lambda^2 \frac{\partial v}{\partial \lambda}$$

This is expression for group velocity.

3.6 Relation between group velocity and phase velocity :

If u be the phase velocity,

$$u = \frac{w}{k} \quad \text{or} \quad w = uk$$

& the group velocity, $V_g = \frac{d\omega}{dk} = \frac{d}{dk}(uk)$

or $$G = u + k\frac{du}{dk}$$

But $$k = \frac{2\pi}{\lambda}$$

$$dk = -\frac{2\pi}{\lambda^2} d\lambda$$

& $$\frac{k}{dk} = \frac{-\lambda}{d\lambda}$$

Therefore group velocity $G = u - \lambda \frac{du}{d\lambda}$

or $$v_g = u - \lambda \frac{du}{d\lambda}$$

This relation shows that group velocity v_g is less than phase velocity u in a dispersive medium i.e. when u is function of λ or k. However in a non-dispersive medium, waves of all wavelengths travel with same speed,

i.e. $\frac{du}{d\lambda} = 0$ then, $v_g = u - \lambda(0)$

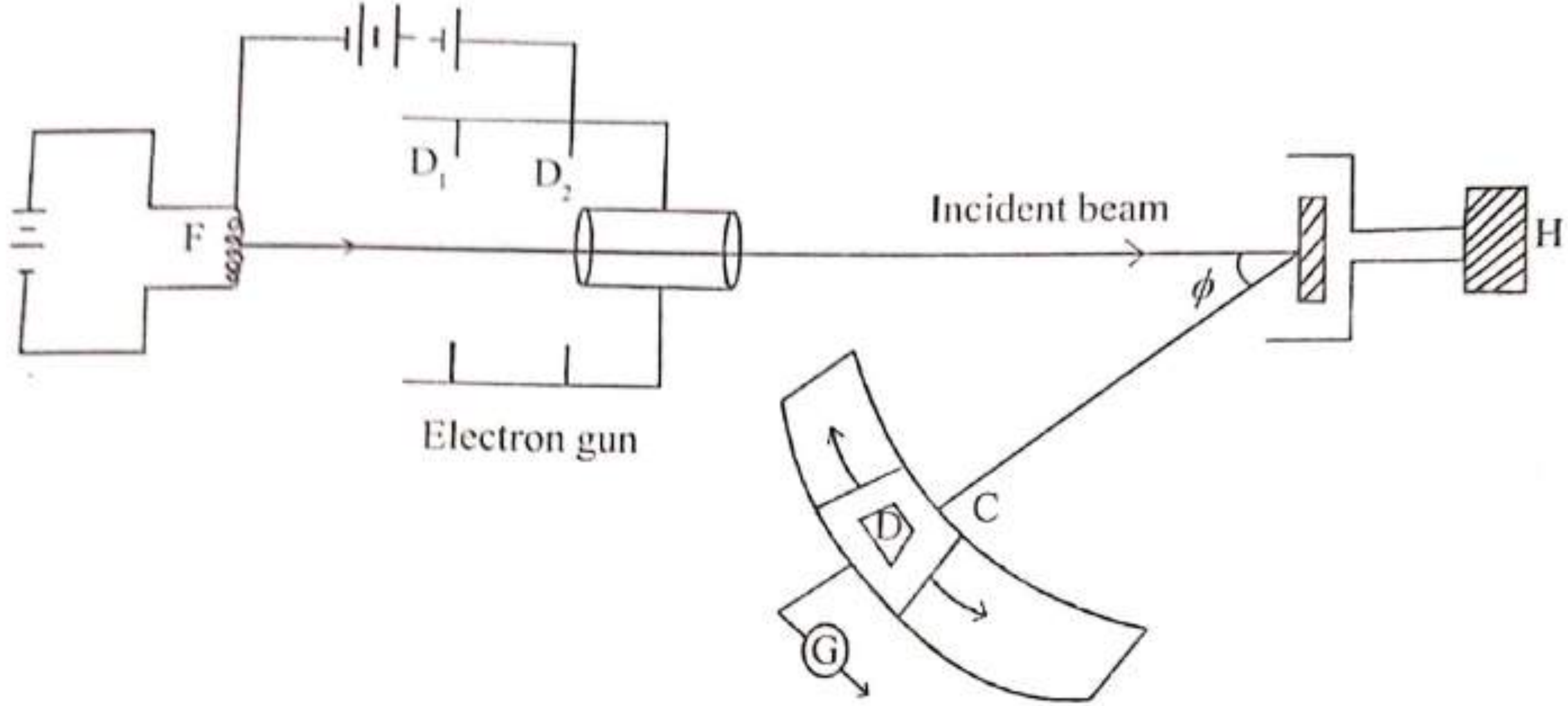
or $v_g = u$ or $u = v_g$

i.e. wave velocity = wave packet velocity.

This is true in electromagnetic wave in vaccum & the elastic waves in homogeneous medium.

3.7 Davission & Germer Experiment :

The wave nature of element was experimentally verified by Davission & Germer in 1927, when they were studying the diffraction of electrons which are diffracted from crystals whose atoms are seprated approximantely. This experimental gave first experimental evidance in support of matter waves. Below fig. shows experimental arrangement,

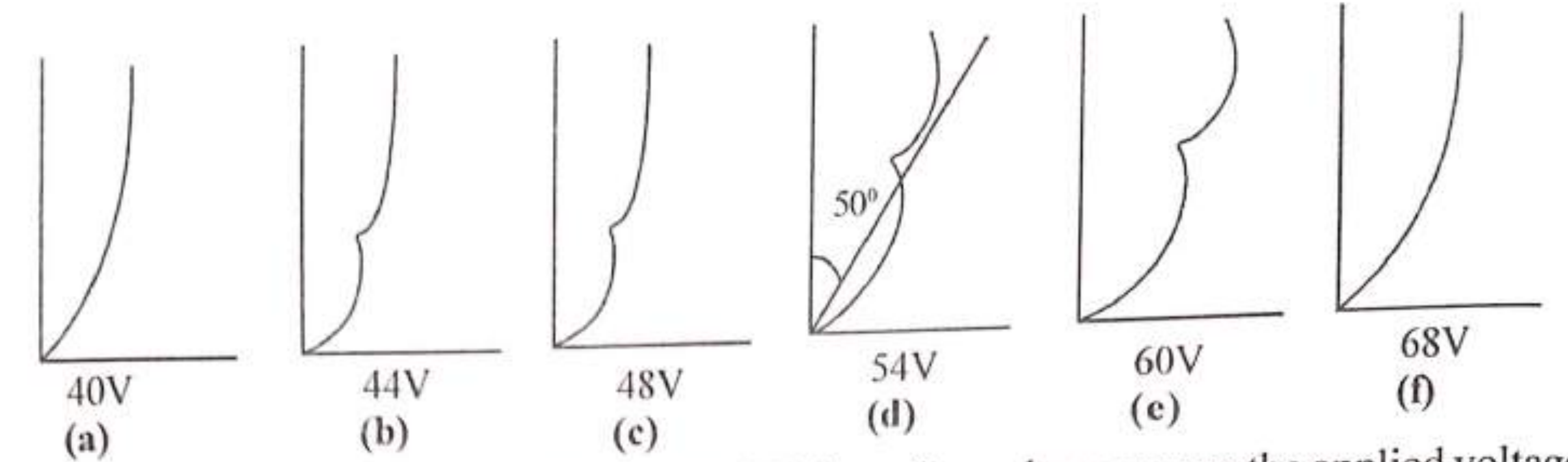


[Fig: Davission & Germer Experiment]

The electrons are produced by heating Fillament F by a low tension battary. The electrons are passed in parallel lines between slits by using aluminium diaphagms D_1 & D_2. These electrons are accelerated by passing them through an aluminium cylinder across which a high potential is applied, this electron beam is allowed to incident on a crystal of nickel target. The crystal can be rotated using handel H about axes parallel to axis of incident beam. The atoms of the crystal scatter the incident electrons in all directions. The scatlered electron are collected by a collector C. Which can moved on a circular scale & connected to a galvonometer. The electrons reflected in all direction making angle 20^0 to 90^0 with the incident beam are collected. C & D are walls of collectors insulated with each other & a retarding potential is applied between the two walls of collectors so that the electrons having velocity equal to that of incident velocity & are exited by collisions with atoms enter the collector. The nickel crystal (Face centered) can be turned by turning the handle so that the reflected beam is allowed to enter the collector.

For Normal incidence:

The beam of electrons is allowed to Fall normally on the surface of crystal. The collector is moved on the circular scale to collect electrons from various directions & galvanometer current is a measure of the intensity of the electron. Below curve plotted by using angles between incident beam & beam entering the collector & galvanometer current for different voltages.



The curve (b) shows bump at 44V applied. Voltage from above curves the applied voltage increases the bump moves upward & has the changes at 54V after that the bump begins to diminish. The bump at 54V or fig (d) shows constructive interference of electron waves scattered in the direction from equally spaced planes of crystals & rich in atoms.

According to De- Broglie,

$$\lambda = \frac{h}{mv}\left[\text{since } \frac{1}{2}mv^2 = eV \;\; or \;\; v = \sqrt{\frac{2eV}{m}}\right] \text{therefore } v^2 = 2eV/m$$

$$\lambda = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} = \frac{6.62\times10^{-34}\,\text{J.sec}}{\sqrt{\left(2\times9.1\times10^{-31}1.6\times10^{-19}\right)V}}$$

$$= \frac{12\cdot 27}{\sqrt{V}} \times 10^{-10} m = \frac{12\cdot 27}{\sqrt{V}} \overset{0}{A}$$

$\therefore$ *For* 54 *volt* $\lambda = \frac{12\cdot 27}{\sqrt{54}} = 1\cdot 67 \overset{0}{A}$

Also for plane reflecting grating,

$2d\sin\theta = n\lambda$ $\quad$ (*For nickel* $d = 2\cdot 15 \overset{0}{A}$)

$\qquad\qquad\qquad\qquad d = 1$

$\therefore \quad 2 \times 2\cdot 5 \sin 50^0 = 1 \times \lambda$

$\therefore \quad \lambda = 1\cdot 65 \, {}^0\text{A}$

Here experimental value of has λ has a close agrement with the theoretcal value.

Hence the electron beam behaves like an X- ray & goes through diffraction at the reflecting surface, thus elelctrons behaves like a wave.

For oblique incidence :

The crystal moved in such way that the angle of incidence is 10^0 & the position of collectors

$$= \frac{12\cdot27}{\sqrt{V}} \times 10^{-10} m = \frac{12\cdot27}{\sqrt{V}} \overset{0}{A}$$

$\therefore$ For 54 volt $\lambda = \frac{12\cdot27}{\sqrt{54}} = 1\cdot67 \overset{0}{A}$

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Here experimental value of has λ has a close agrement with the theoretcal value.

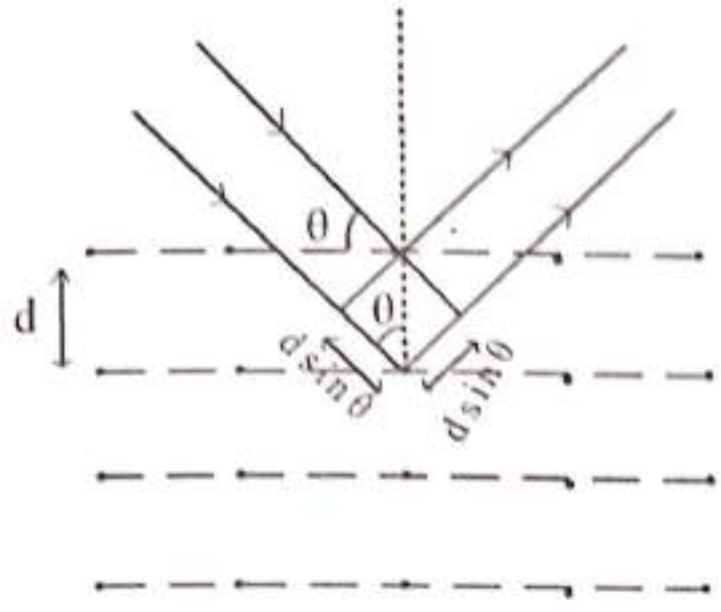
Hence the electron beam behaves like an X- ray & goes through diffraction at the reflecting surface, thus elelctrons behaves like a wave.

For oblique incidence :

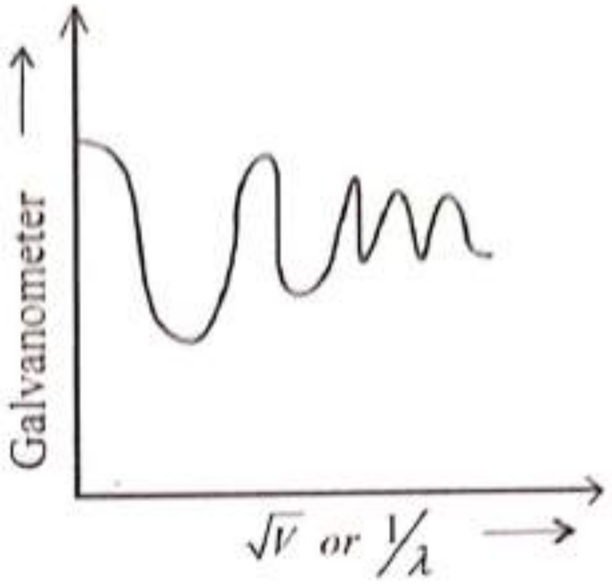
The crystal moved in such way that the angle of incidence is 10^0 & the position of collectors

corresponds to angle of reflection 10^0. By applying the alternating voltage V, the electrons undergo Bragg's type reflection & Follows the condition $2d\sin\theta = n\lambda$.

The electrons are diffracted by successive parallel planes of atoms of crystal. The below graph of current & $\sqrt{V}$ *or* $1/\lambda$ gives relation between current & $1/\lambda$, shown in fig.



[Fig: Braggs types reflection]



[Fig: Relation between current & $\frac{1}{\lambda}$]

By using De- Broglie relation,

$$\lambda = \frac{h}{\sqrt{2meV}} \quad ie \quad \lambda \propto \frac{1}{\sqrt{V}}$$

The Braggs relation status that,

$$2d\sin\theta = n\lambda = \text{constant}$$

or
$$n = \frac{\text{constant}}{\lambda} = \frac{\text{constant}}{h/\sqrt{2meV}}$$

$$n = \frac{\text{constant} \times \sqrt{2me}.\sqrt{V}}{h}$$

$\therefore\ n \propto \sqrt{V}$ i.e order of spectrum proportional to square root of applied voltage V.

3.8 Heisenberg uncertainty Principle:

In 1927, Heisenbergs proposed the uncertainty principle which states that "The simultaneous determination of the exact position & momentum of a moving particle is impossible.

It is best known result of the wave particle duality i.e. concept of wave or wave packet associated with a moving particle.

The quantity $\left|\psi(x,t)\right|^2 \Delta x$ represents the probability that the particle is within the range or region between x & x + Δx . It means that there is an uncertainty in the location of the position of the particle & Δx is measure of uncertainity. The uncertainity in the position would be less if Δx is smaller, i.e. if wave packet is very narrow. The narrow wavepacket means the range of wavelength $\Delta\lambda$ between λ & $\lambda+\Delta\lambda$ is smaller or range of wavenumber Δk between k & Δk is larger. So Δx is inversally proportional to Δk i.e.

$$\Delta x \propto \frac{1}{\Delta k}$$

We may approximate this as $\Delta x \Delta k \cong 1$

Taking $\hbar = \frac{h}{2\pi}$ we get, $p = \frac{h}{\lambda}$

$$p = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda}$$

$$= \hbar k$$

$$\Delta k = \frac{\Delta p}{\hbar}$$

Therefore, $\Delta x \Delta p \cong \hbar$

The above relation represents the lowest limit of accuracy. Therefore, we can write,

$\Delta x \Delta p \geq \hbar$

The principle of uncertainity can also be represented in terms of energy E & time t.

Since $$\frac{\Delta p}{\Delta t} = \Delta F$$

We can write, $$\frac{\Delta (mv)}{\Delta t} = \Delta F$$

or $$\Delta p = \Delta F \times \Delta t$$

putting this value of Δp in the expression $\Delta x \Delta p \geq \hbar$ we obtain,

$$\Delta x \times (\Delta F \times \Delta t) \geq \hbar$$

$$[\Delta F \times \Delta x]\ \Delta t \geq \hbar$$

or $$\Delta E\ \Delta t \geq \hbar$$

The principle of uncertainity can also be expressed interms of angular momentum & angular displacement suppose we have a particle at a particular angular position and its angular momentum is $L\theta$. Then the limits in the uncertainity $\Delta\theta$ & $\Delta L\theta$ are given by the relation $\Delta\theta\ \Delta L\theta \geq h$.

Mathematical Proof :

According to DeBroglie's wave concept that a material particle in motion is equivalent to a group of waves or wave packet, the group velocity G being equal to the particle velocity V. Consider a simple case of wavepacket which is formed by the superposition of two simple harmonic plane waves of equal amplitudes and having equal frequencies ω_1 & ω_2. The two waves can be represented by the equations.

$$y_1 = a \sin(\omega_1 t - k_1 x)$$

$$y_2 = a \sin(\omega_2 t - k_2 x)$$

Where k_1 & k_2 are propagation constant & $\frac{\omega_1}{k_1}$ $\frac{\omega_2}{k_2}$ are their respective phase velocities.

Then the resultant wave due to superposition of these wave is given by,

$y = y_1 + y_2$

$$y = 2a \sin(\omega t - kx) \cos\left[\frac{\Delta w}{2}t - \frac{\Delta k}{2}x\right]$$

Where $w = (w_1 + w_2)/2$

$k = (k_1 + k_2)/2$

$\Delta\omega = \omega_1 - \omega_2$ & $\Delta k = k_1 - k_2$.

Below fig shows resultant wave, the envelope (loop) of this wave travels with the group velocity G i.e.

$$\Delta x = \frac{2\pi h}{2\pi \Delta p}$$

= $$\therefore \ \Delta x = \frac{h}{\Delta p} \quad \text{or} \quad \Delta p \, \Delta x = h$$

However more accurate measurements show that the product of uncertainities in momentum Δp & position Δx cannot be less than $h/2\pi$.

Therefore,

or $\Delta p \, \Delta x \geq h$

This is the Heisenberg's uncertainity principle.

Uncertainty in time and energy:

Consider the electron of mass m moving with velocity v.

The Kinetic energy of electron is,

$$E_k = \frac{1}{2} m v^2$$

Taking mass of electron constant then,

$$\Delta.E_k = \Delta.\left(\frac{1}{2}mv^2\right) = mv.\Delta v = v\Delta p$$

Now the velocity $v = \dfrac{\Delta x}{\Delta t}$

$$\Delta.E_k = \frac{\Delta x}{\Delta t}.\Delta p$$

$$\Delta x.\Delta p = \Delta.E_k.\Delta t \geq \frac{1}{2}\frac{h}{2\pi} = \frac{1}{2}\hbar$$

$$\Delta.E_k.\Delta t = \frac{1}{2}\hbar$$

This equation shows uncertainty in time and energy.

Uncertainty in angular momentum and angular position:

Consider a particle of mass m moving in a circle of radius R with velocity v. As particle moves in a circular path the uncertainty principle apply in a direction tangent to the circle,

$$\Delta P_s \Delta s = \frac{\hbar}{2}$$

Where s is the linear displacement and P_s is linear momentum

Now angular displacement θ is related to arc lenth of s then,

$$\theta = \frac{s}{R} \quad \Delta s = R.\Delta\theta$$

The angular momentum L is rotated to linear momentum by,

$$L = mvR = P_s R$$

$$\Delta P_s \Delta s = \frac{\Delta L}{R} R.\Delta\theta$$

Hence, $\Delta P_s = \dfrac{\Delta L}{R}$

therefore $\Delta P_s.\Delta s = \dfrac{\hbar}{2} = \Delta L.\Delta\theta = \dfrac{\hbar}{2}$

Hence proved.

3.9 Application of Uncertainty principle :

(1) Non exsistance of electron in nucleus:

If the electron exist inside the nucleus then uncertainity in its position. The energy must be of the order of 97MeV. But experimentally observed that no electron in the atom possesses energy greater than 4MeV. Obviously we conclude that electrons do not reside in the nucleus.

The radius of a atomic necleus is of the order of , Hence if the electron is to exsist within the necleus the uncertainty in its position then,

$$\Delta x \geq 10^{-14} m$$

According to Heisenbergs uncertainity principle,

$$\Delta x.\Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{6.6\times10^{-34}}{2\times2\pi\times10^{-14}} = \frac{6.6\times10^{-34+14}}{12.56}$$

$$p = 0.53\times10^{-20}$$

$$p = 5.3\times10^{-21} Kg.m/s$$

Means the total momentum of the electron in the nucleus must be at least of the order

$$p = 5.3 \times 10^{-21} Kg.m/s$$

Now the kinetic energy of the electron of mass m is given by,

$$E_k = \frac{p^2}{2m} = \frac{\left(5.275 \times 10^{-21}\right)^2}{2 \times 9.1 \times 10^{-31}}$$

$$= \frac{27.8256 \times 10^{-42+31}}{18}$$

$$= \frac{1.5458 \times 10^{-11}}{1.6 \times 10^{-19}}$$

$$= 0.97 \times 10^{-11+19} = 0.97 \times 10^{8} = 97 MeV$$

Means if electron exist inside the nucleus energy must be of the order 97MeV. But experimental observations show that no electron in the atom possesses energy greater than 4MeV. Hence electrons do not reside in the nucleus.

(2) Binding energy of an electron in an atom :

Consider the electron revolving round the nucleus of an atom in an orbit of radius r. As the electron is inside the atom it is located within a distance r. The uncertainty in the position of the electron is Δx equal to its radius of orbit.

$$\Delta x = r$$

According to uncertainty principle

$$\Delta x.\Delta p = \frac{\hbar}{2} \qquad \Delta p.r = \frac{\hbar}{2}$$

$$\Delta p = \frac{\hbar}{2.r}$$

Hence minimum value of momentum of the electron in its orbit is

$$\rho = \Delta p = \frac{\hbar}{2.r}$$

Taking r to be of the order of $10^{-10} m$ then the value of momentum

$$p = \frac{\hbar}{2.r} \quad \therefore \hbar = \frac{h}{2\pi} \quad p = \frac{h}{2\pi \times 2r} = \frac{h}{4\pi.r}$$

$$\therefore p == \frac{h}{4\pi.r} = \frac{6.6 \times 10^{-34}}{4\pi \times 10^{-10}} = 5.2 \times 10^{-25} Kg.m/s$$

As the momentum is non relativistic, the kinetic energy of the electron

$$k = \frac{p^2}{2m} = \frac{\left(5.2 \times 10^{-25}\right)^2}{2 \times 9.1 \times 10^{-31}} = 1.5 \times 10^{-19} J = 1eV\,(Approx.)$$

Now potential energy of electron,

$$V = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$V = \frac{-Z\left(1.6 \times 10^{-19}\right)^2}{4 \times 3.14 \times 8.56 \times 10^{-12} \times 10^{-10}}$$

$$But,\ \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\times 3.14\times 8.56\times 10^{-12}} = 0.0093\times 10^{12} = 9.3\times 10^{9}$$

$$V = \frac{-Z.\left(1.6\times 10^{-19}\right)\times\left(1.6\times 10^{-19}\right)\times\left(9.3\times 10^{9}\right)10^{10}}{\left(1.6\times 10^{-19}\right)}$$

$$= -Z.\left(1.6\times 10^{-19}\right)\times\left(9.3\times 10^{9}\right)$$

$$= -Z\times 14.4eV$$

Therefore $E = K + V = -13.4ZeV$.

For hydrogen Z=1

$\therefore$ E=1-14.4=-13.4eV

For helium Z=2

$\therefore$ E=1-28.8=-27.8eV

This two values agree closely with binding energy of outermost electron in hydrogen (-13.4eV) and helium (-27.8eV) respectively and roughly it is 15 eV.

:: Multiple Choice Questions ::

1) Which represents De Broglies wavelength .....

a) $\lambda = \frac{h^2}{m\upsilon}$ b) $\lambda = \frac{\hbar}{m\upsilon}$

c) $\boldsymbol{\lambda = \frac{h}{m\upsilon}}$ d) $\lambda = \frac{h}{m\upsilon^2}$

2) The De Broglies hypothesis is associated with ....

a) Wave nature of electrons only

b) Wave nature of particles

c) wave nature of radiation

d) Wave nature of all material particles

3) Which is not a uncertainty principle ....

a) $\Delta x.\Delta p \geq \hbar$ b) $\Delta t.\Delta E \geq \hbar$

c) $\Delta\theta.\Delta J \geq \hbar$ **d)** $\boldsymbol{\Delta p \geq \Delta x\hbar}$

4) The relation between phase velocity u and group velocity υ_g is ....

a) $u.\theta_g = c$ b) $\upsilon_g = c^2 u$

c) $\boldsymbol{u.\upsilon_g = c^2}$ d) $u = c^2\upsilon_g$

5) Matter waves are ....

a) Longitudinal b) electromagnetic

c) alwaystravel with speed of light **d) show diffraction**

6) If a charge particle of mass m is accelerated through a potential difference of v volts. The De Broglies wavelength is propotional to ....

a) v b) $\sqrt{v}$

c) v^2 **d)** $\mathbf{v^{-1/2}}$

7) The De Broglies wavelength of material particles which are in thermal equillibrium at temperature T is ....

a) $\frac{h}{\sqrt{2mKT}}$
b) $\frac{h}{\sqrt{2KT}}$
c) $\frac{\hbar}{\sqrt{2mKT}}$
d) $\frac{\hbar}{\sqrt{mKT}}$

8) Which of the following pairs of phenomenon illustrates the particle aspect of wave particle duality ....

a) Compton effect and Braggs law
b) Braggs low and Photoelectric effect
c) Photoelectric effect and compton effect
d) Compton effect and Pauli's principle

9) uncertainty relation holds for, ...

a) microscopic particles only
b) macroscopic particle
c) microscopic and macroscopic both
d) none of these

10) The duration of radar pulse is 10^{-6}. The uncertainty in its energy would be ....

a) $1.05 \times 10^{-21} J$
b) $1.05 \times 10^{-35} J$
c) 1.05×10^{-28} J
d) 0

11) The uncertainty principle cannot hold for the following pairs ....

a) Angular momentum and angle
b) Linear momentum and angle
c) energy and time
d) position and momentum

12) The De Broglies wavelength λ of an electron in n^{th} Bohr orbit is related to the radius r of the orbit is ....

a) $n\lambda = \pi r$
b) $n\lambda = 2\pi r$
c) $n\lambda = 4\pi r$
d) $n\lambda = \frac{3}{2}\pi r$

13) De Broglies wavelength of a particle is ....

a) proportional to its mass
b) Proportional to its energy
c) Proportional to momentum
d) Inversaly propostional to its momentum

14) The De Broglies wavelength is independent of ....

a) Mass
b) charge
c) velocity
d) momentum

15) Devission and Germar experiment is related to ....

a) Interfernce
b) Polarization
c) Diffraction
d) All of the Above

16) The concept of duality is firstly proposed by ....

a) De Broglies
b) Einstein
c) Taylor
d) G. P. Thomson

17) Which of the following relation is true ....

a) $\lambda \propto v$
b) $\lambda \propto v^2$
c) $\lambda \propto \frac{1}{\sqrt{v}}$
d) $v \propto \lambda$

18) Which relation is correct ....

a) $v = n\lambda$
b) $k = \frac{2\pi}{\lambda}$
c) $v_g = \frac{\partial w}{\partial k}$
d) All of these

19) The De Broglies wavelength of material particles which are in thermal equillibrium at temperature T is ....

a) $\frac{h}{\sqrt{3mKT}}$
b) $\frac{\hbar}{\sqrt{2mKT}}$
c) $\frac{\hbar}{\sqrt{mKT}}$
d) $\frac{h}{\sqrt{3KT}}$

20) The phase velocity and group velocity of De Broglies wave in free space are related as

a) $\frac{V_p}{V_g} = \sqrt{2}$
b) $\frac{V_p}{V_g} = 0.5$
c) $V_p V_g = c^2$
d) $V_p V_g = \sqrt{2}c^2$

21) The centipetal force between electron in a bohr orbit of radius r and nucleus of charge is

a) $\frac{Z_e^2}{4\pi\varepsilon_0 r}$

b) $\frac{Z_e^2}{4\pi\varepsilon_0 r}$

c) $\frac{Z_e^2 e^2}{4\pi\varepsilon_0 r}$

d) $\frac{Z_e^2 e^2}{4\pi\varepsilon_0 r^2}$

22) Every moving material particle has a .... nature.

a) Single

b) **dual**

c) both (a) and (b)

d) none of these.

23) The De Broglies wavelength is ....proportional to the momentum of the particle.

a) **inversaly**

b) directly

c) constant

d) none of these.

24) $\Delta J . \Delta\theta \geq \hbar$ represents .... principle.

a) Bohr

b) Davission and Germer

c) **Heisenbergs uncertainty**

d) None of these

THANK YOU