F.Y.B.SC. PHYSICS

First Semester Paper II

Heat and Thermodynamics

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Chapter - 3

Thermodynamics

3.1 Adiabatic Process :

Def : "When a change in pressure and volume of a substance takes place but no heat is allowed to enter or leave it, the change or process is called adiabatic."

If no heat is taken away in the first case the temperature will increase and if no heat is supplied in second case the temperature will fall.

Hence in an adiabatic change, the temperature does not remain constant and no heat from outside is supplied to the system or taken away from it. Means heat content of the system remains constant. when gas is allowed to expand, it does external work and the energy for this purpose is drawn from the internal energy of the gas. The internal energy of the gas therefore decreases. When the gas compressed, work is done on it and the energy gained appears as internal energy. The internal energy of the gas therefore increases.

During an adiabatic no heat enters or leave the gas,

$$\therefore \quad \delta Q = 0$$

According to the first law of thermodynamics,

$$\delta Q = dU + \delta W$$

$$\therefore \quad 0 = dU + \delta W$$

Or
$$\boxed{dU = -\delta W}$$

$\therefore$ Increase or decrease in internal energy = External work done on the gas.

From above,

When the volume of a substance increases (or pressure decreases) in an adiabatic change the process is known as **adiabatic expansion**.

Also when the pressure of a substance increases (or volume decreases) in an adiabatic change the process is known as adiabatic compression.

3.2 Adiabatic equation of a perfect gas :

Consider a one gm molecule of a perfect gas contained in a perfectly non-conducting cylinder fitted with non-conducting piston shown in below fig.
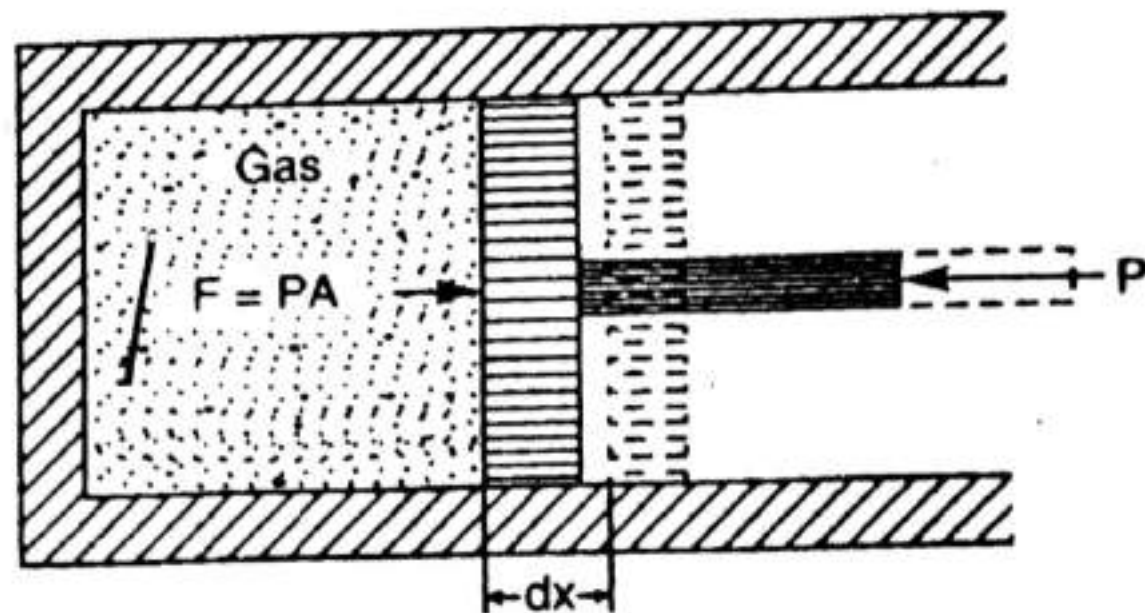
[Fig : Perfect gas contained in non-conducting cylinder]

Let the pressure, volume and temperature be P, V and T respectively. Suppose the gas is compressed adiabatically, so that piston moves inwards through a distance dx. If A is area of the cross section then,

Total force applied is PxA.

Work done by piston = Force x distance

$$dW = P \times A \times dx.$$

or $\quad\quad\quad\quad\quad dW = PdV \quad\quad\quad\quad\quad (Adx = dV)$

According to first law of thermodynamics,

$$dQ = dV + dU$$

$$dQ = dV + PdV$$

For an adiabatic change $dQ = 0$.

$$\therefore dU + PdV = 0$$

Now $\quad C_V = \dfrac{dU}{dT}$

Here C_V is gram molecular specific heat at constant volume.

$$\therefore dU = C_V dT$$

Hence $C_V dT + PdV = 0$ \quad\quad\quad\quad ------ (1)

The equation of state for one mole of a perfect gas is $PV = RT$ \quad ------ (2)

Differentiating equation (2) we have,

$$PdV + VdP = RdT.$$

Or $\quad dT = \dfrac{PdV + VdP}{R}$

Substituting the value of dT in (1) we have,

$$C_V = \dfrac{PdV + VdP}{R} + PdV = 0$$

Or $\quad C_V PdV + CvVdP + RP\ dV = 0$

Or $\quad (C_V + R) PdV + C_V VdP = 0$

But $C_P - C_V = R$ Or $C_V + R = C_P$

Where C_P is the gram molecular specific heat at constant pressure.

Hence $C_P PdV + C_V VdP = 0$

Dividing $C_V P_V$ and putting $\dfrac{C_P}{C_V} = \gamma$

Where γ is ratio of specific heat at constant pressure to specific heat at constant volume

we have,

$$V\frac{dV}{V} + \frac{dP}{P} = 0$$

Integrating above equation we get,

$$\int \gamma \frac{dV}{V} + \int \frac{dP}{P} = 0$$

$$\gamma \log_e V + \log_e P = \text{constant.}$$

Or $\qquad \log_e PV^\gamma = \text{constant}$

$$PV^\gamma = e^{\text{constant}} = \text{constant} = K$$

Thus $\boxed{PV^\gamma = \text{constant}}$ ----- (3)

From eqⁿ (1)

$$C_V dT + PdV = 0$$

Put $P = \dfrac{RT}{V}$ & divide by C_V, T

then,

$$\frac{dT}{T} + \frac{R}{C_V}\frac{dV}{V} = 0$$

Integrating we get,

$$\int\frac{dT}{T} + \frac{R}{C_V}\int\frac{dV}{V} = 0$$

Or $\quad \log_e T + \dfrac{R}{C_V}\log_e V = \text{constant}$

Or $\quad \log_e T_V + R/C_V = \log_e TV^{(C_P-C_V)/C_V}$

$$\log_e T_V + R/C_V = \log_e TV^{\gamma-1} = \text{constant}$$

Or $\quad \boxed{TV^{\gamma-1} = \text{a constant}}$ ------ (4)

This equation is equation of state for an adiabatic change in terms of thermodynamics variables T and V.

Now, for perfect gas,

$$PV = RT$$

Or $\quad V = \dfrac{RT}{P}$

Substituting V in the relation $PV^{\gamma} = K$

we have,

$$P\left(\dfrac{RT}{P}\right)^{\gamma} = K$$

Or $\quad \dfrac{T^{\gamma}}{P^{\gamma-1}} = \dfrac{K}{R^{\gamma}} = K_1$

Or $\quad T^{\gamma}\, P^{\gamma-1} = \text{constant}$

Or $\quad \boxed{T\, P^{(1-\gamma)/\gamma} = \text{constant}}$ \quad ------ (5)

equation (5) shows equation of state for an adiabatic change in terms of variables P & T. This is the equation of state for an adiabatic change.

If P_1V_1 are the initial & P_2V_2 be the final pressure and volume of the gas in an adiabatic change then,

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

Below plot of volume and pressure shows adiabatic expansion curve AB. The initial state $A(P_1V_1T_1)$ is carried out in such way that no heat enters or leaves the system, finally it reaches to point $B(P_2, V_2, T_2)$ in which decrease in temperature $(T_2 < T_1)$
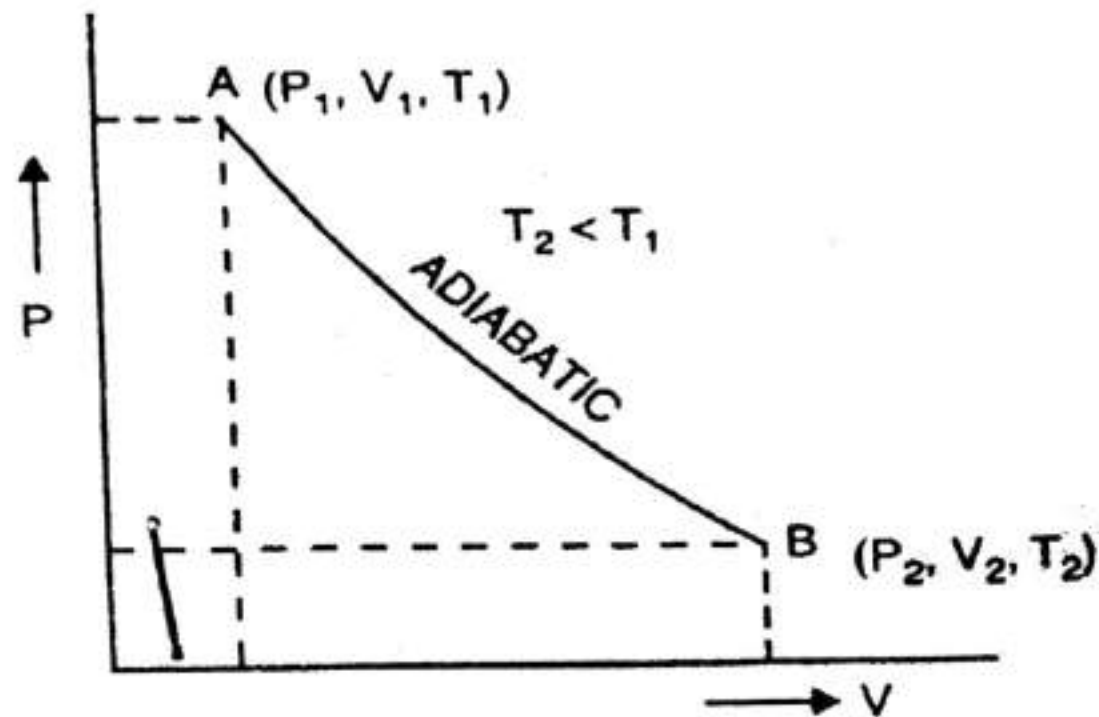
Adiabatic relation between V and T :

From equation (1)

$$C_V dT + PdV = 0$$

But $P = \dfrac{RT}{V}$ & divide by $C_V T$ we get,

$$\dfrac{dT}{T} + \dfrac{R}{C_V} \dfrac{dV}{V} = 0 \quad \text{Integrating we have,}$$

Or

$$\boxed{\int \dfrac{dT}{T} + \int \dfrac{R}{C_V} \int \dfrac{dV}{V} = 0}$$

3.3   Isothermal Process :

Defination : "When a change in pressure and volume of a substance takes place but the temperature remains constant, the change or process is called as isothermal process."
Also when a gas is compressed suddenely, some heat is produced but if the compression is slow and the heat produced is removed at once, so that the temperature remains constant, the change is isothermal.

Similarly when a gas is allowed to expand suddenly, work is done by the gas and some heat is absorbed. If heat is continuously supplied from outside so that the temperature remains constant the change is isothermal.

Thus, in an isothermal change the temperature is kept constant by adding heat or taking it away from the substance. As there is no change in temperature, there is no change in internal energy.

Therefore $dU = 0$.

According to first law of thermodynamics,

$$\delta Q = dU + \delta W$$

where δQ and δW are not perfect differentials but du is perfect differential because U is function of the state of the system.

$$\delta Q = 0 + \delta W$$

Or $\qquad \delta Q = \delta W$

Therefore in isothermal process, Heat added or subtracted = External work done by the gas.

In below fig. the working substance undergoes isothermal expansion, starting from point A having initial pressure P_1 & volume V_1.
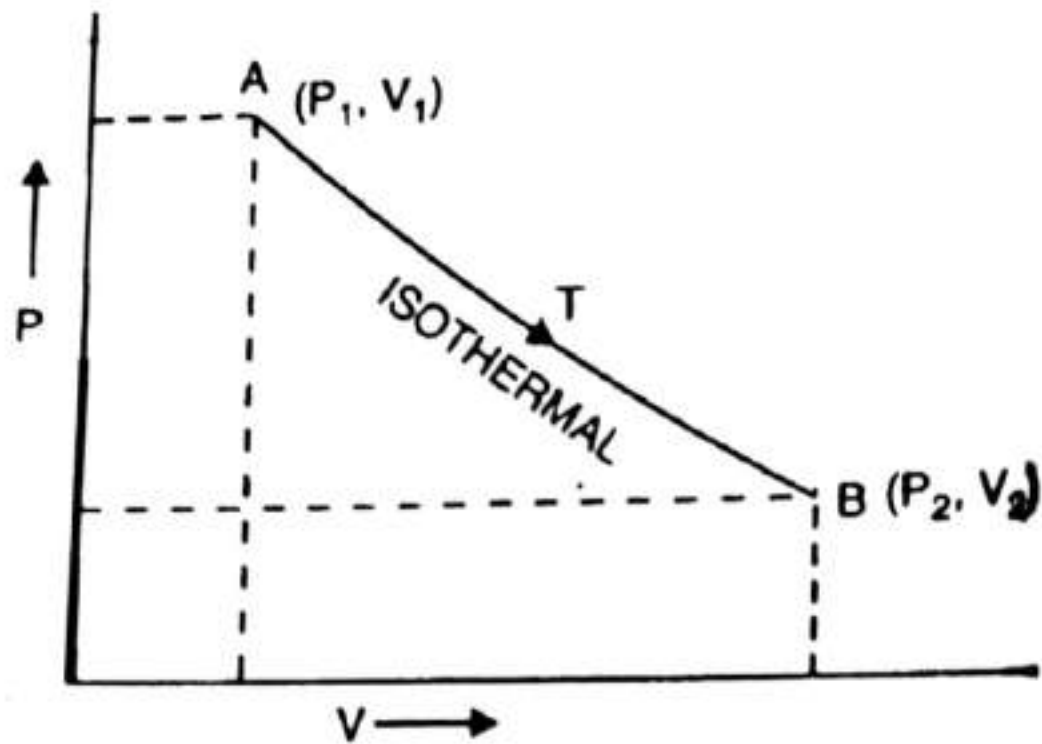
[Fig : Isothermal expanssion curve]

Pressure is decreased from A to B and work is done by the working substance at the cost of its internal energy and suffers a fall in temperature. But the system is perfectly conducting to the surroundings. It absorbs heat from the surroundings and maintain a constant temperature.

Thus, from A to B the temperature remains constant and the curve AB is called **isothermal curve** or **isothermal**. In going from B to A back, the system gives out extra heat to the surroundings and maintains the temperature constant

Now, for isothermal process,

$PV = RT = $ constant.

For n gram molecules of a gas,

$PV = nRT$

For an ideal gas,

$$(U_2 - U_1) = 0$$

$\therefore$ From the first law of thermodynamics, we get,

$$\boxed{Q = W}$$

i.e. in an isothermal process the heat supplied to an ideal gas is equal to the work done by the gas.

3.4 Indicator diagram :

Indicator diagram is very useful to represent the behaviour of an engine. This helps to understand the performance of heat engines.

Defination : In equillibrium state each point on the path traced represents the state of the system in terms of co-ordinates (P, V). Such trace obtained in an actual engine is called indicator diagram or P - V diagram.
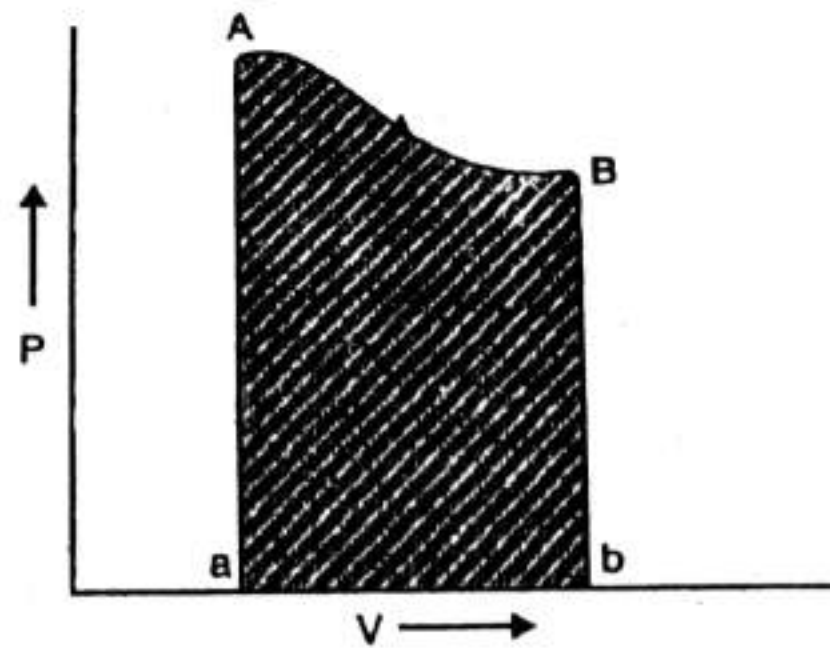
[Fig : Indicator diagram]

In above diagram each point on the path traced represents the state of the system in terms of co-ordinates (P.V). Such trace obtained in an actual engine is called indicator diagram or P-V diagram.

Now the work done $W = \int PdV$

$$W = \text{area ABba}$$

Here, the gas in expanding against. Pressure P is PdV. dV is the infinitesimal change in volume. Hence, the indicator diagram directly indicates the work done by the engine during each

cycle of operation. The work being equal to the area enclosed by the indicator diagram. The workdone is +ve if the indicator diagram is traced clockwise direction. If the direction is anticlockwise the work done is -ve.

3.5 a : Work done during an isothermal process :
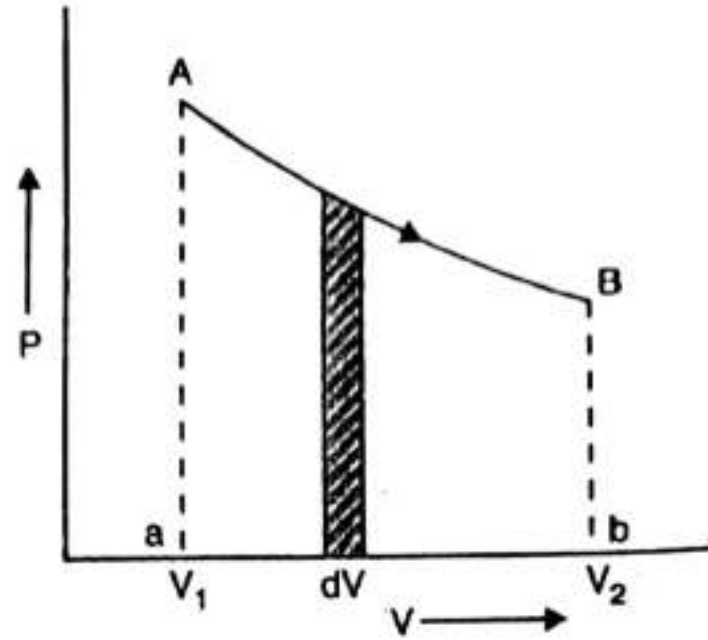
[Fig. : Work done during an isothermal process]

When a gas is allowed to expand isothermally, work is done by it.

In this process, the change in the internal energy of the system is zero because of the constant temperature. So the heat absorbed by the system is equal to the work done by it.

Let V_1 and V_2 be the initial and final volumes respectively shown in fig. or indicator diagram. The area of shaded strip represents the work done for a small change in volume dV. when volume changes from V_1 to V_2 then,

volume changes from V_1 to V_2 then,

work done $W = \int_{V_1}^{V_2} PdV$

i.e. $W = $ Area ABba

Considering one gram molecule of the gas

$PV = RT$

$\therefore P = \dfrac{RT}{V}$

Therefore above equation becomes,

$W = RT \int_{V_1}^{V_2} \dfrac{dV}{V}$

$W = RT \times 2.303 \log_{10} \dfrac{V_2}{V_1}$

Also $\quad P_1 V_1 = P_2 V_2$

Or $\quad \dfrac{V_2}{V_1} = \dfrac{P_1}{P_2}$
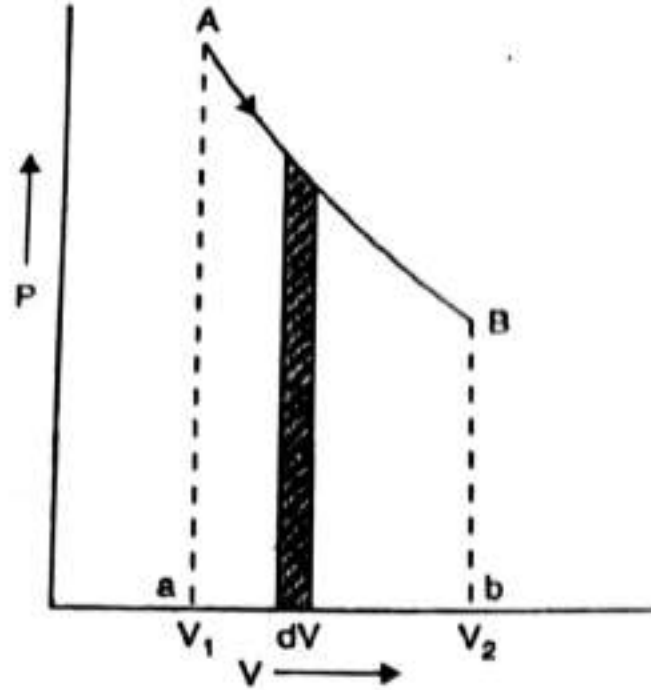
$$\therefore W = RT \times 2.303 \log_{10} \dfrac{P_1}{P_2}$$

----- (3)

3.5 b : Work done during an abiabatic process :

The work done in adiabatic process depends only upon the initial and final temperatures T_1 and T_2.

During an adiabatic process, the system is thermally insulated from the surroundings. The gas expands from volume V_1 to V_2 as shown in indicator diagram.

[Fig. : Work done during an adiabatic process]

The work done by the gas for an increase in volume dV is PdV.

Now work done, when the gas expands from V_1 to V_2 is given by,

$$W = \int_{V_1}^{V_2} PdV = \text{Area ABba}$$

During an adiabatic process,

$$PV^\gamma = \text{constant} = K$$

Or $$P = \frac{K}{V^\gamma}$$

$$W = K \int_{V_1}^{V_2} \frac{dV}{V^\gamma}$$

$$W = \frac{K}{1-\gamma} \left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right]$$

Since A & B lie on the same abiabatic

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$W = \frac{1}{1-\gamma} \left[\frac{K}{V_2^{\gamma-1}} - \frac{K}{V_1^{\gamma-1}} \right]$$

$$W = \frac{1}{1-\gamma} \left[\frac{P_2 V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1 V_1^\gamma}{V_1^{\gamma-1}} \right]$$

$$W = \frac{1}{1-\gamma} \left[P_2 V_2 - P_1 V_1 \right] \qquad \text{------ (1)}$$

Or $\qquad W = \frac{1}{1-\gamma} \left[P_1 V_1 - P_2 V_2 \right]$

Taking T_1 and T_2 as the temperatures at points A and B respectively. Considering one gram molecule of the gas,

$$P_1 V_1 = RT_1$$

$$P_2 V_2 = RT_2$$

Put these values in equation (1) we have,

$$W = \frac{1}{1-\gamma}\left[RT_2 - RT_1\right]$$

$$\boxed{W = \frac{R}{\gamma - 1}\left[T_1 - T_2\right]}$$

Thus the work done during in adiabatic process depends only initial and final temperatures. Hence the work done along any adiabatic between two isothermals is independent of the particular adiabatic.

$$W = K \int_{V_1}^{V_2} \frac{dV}{V^\gamma}$$

$$W = \frac{K}{1-\gamma} \left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right]$$

Since A & B lie on the same abiabatic

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$W = \frac{1}{1-\gamma} \left[\frac{K}{V_2^{\gamma-1}} - \frac{K}{V_1^{\gamma-1}} \right]$$

$$W = \frac{1}{1-\gamma} \left[\frac{P_2 V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1 V_1^\gamma}{V_1^{\gamma-1}} \right]$$

$$W = \frac{1}{1-\gamma} [P_2 V_2 - P_1 V_1] \qquad \text{------ (1)}$$

Or $\qquad W = \dfrac{1}{1-\gamma} [P_1 V_1 - P_2 V_2]$

Taking T_1 and T_2 as the temperatures at points A and B respectively. Considering one gram molecule of the gas,

$$P_1 V_1 = RT_1$$

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Put these values in equation (1) we have,

$$W = \frac{1}{1-\gamma} [RT_2 - RT_1]$$

$$\boxed{W = \frac{R}{\gamma-1} [T_1 - T_2]}$$

Thus the work done during in adiabatic process depends only initial and final temperatures. Hence the work done along any adiabatic between two isothermals is independent of the particular adiabatic.

3.6   Reversible and Irreversible processes :

a)   Reversible Process :

A reversible process is that which can be retraced in the opposite direction, so that the working substances passes through exactly the same condition as it does in the direct process.

Or it means that in a step where heat is absorbed in the direct process, it is given out in the reverse process & vice versa.

Similarly in a step where work is done by the working substance in the direct process an equal amount of work is done on the working substance in the reverse process.

Consider a gas enclosed in a cylinder fitted with a frictionless pistion, perfectly insulated from its surroundings so that no heat can leave or enter into it.

Some removable weights are placed on the top of the pistion and P_1, V_1, T_1 are the pressure, volume and temperature of the gas in the cylinder in the initial state.

Now the weights are removed by very small amount at a short time so that the piston moves outwards very very slowly.

The gas now expands adiabatically and the pressure and volume of the gas adjust themselves so that the system is always in thermodynamic equillibrium. OR The adiabatic expansion of the gas is quasistatic. The process of removing the weights in small steps is continued so that P_2, V_2, T_2 are the pressure, volume and temperature of the gas in the final state.

Now the weights are again replaced in small amounts at a time so that the piston moves inwards very very slowly. Means the adiabatic compression of the gas is also quasistatic. The process of replacing the weights in small steps is continued still the gas reaches the initial state with P_1, V_1, T_1 as pressure volume and temperature. During this process of adiabatic compression the gas would pass through exactly the same intermediate steps through which it passed during the adiabatic expansion.

i.e. All the intermediates states are retraced in the opposite direction, such a process is known as reversible process. **Hence, A reversible process is that which can be retraced in the opposite direction so that the working substances passes through exactly the same condition as it does in the direct process.**

Examples :

1) All isothermal and adiabatic changes are reversible when performed slowly. Assume that there is no friction to be overcome as work done in overcoming friction is wasted. The process carried out is very slow, hence no energy is wasted in producing oscillations and eddy currents and no heat is lost by conduction, convection and radiation. In this case if heat is supplied to a Diven mass of a gas at constant pressure, it expands & does some extermal work. If the same amount of work is done on the gas in compressing it, the same quantity of heat will be given out.

2) Ice melts, when a certain amount of heat is absorbed by it. The water so formed can be

converted into ice if the same amount of heat is removed from it.

3) If the resistance of a thermocouple is neglected there will be no heat produced due to Joule hating effect, then Peltier heating or cooling is reversible.

4) Electrolysis.

b) **Irreversible Process :**

An irreversible process is that which cannot be retraced in the opposite direction so that the working substance does not pass through the same intermediate states through which it passes during the direct process.

If the weights on the piston are removed quickly the gas will undergo sudden expanssion so that the intermediate states will not be equilibrium state i.e. it will not be possible to define the value of pressure, volume and temperature for these states.

Similarly when the weights are replaced quickly the gas will undergo sudden compression and the value of pressure volume and temperature for the intermediate, States will again be not defined.Thus during the process of sudden compression it will not be possible to in reproduce the intermediate states through which the system passed during sudden expansion, such process is known as irreversible process.

Examples :

1) Joule - Kelvin effect is irreversible because on reversing the flow of gas a similar cooling or heating effect is not observed. Similarly diffusion, conductions and radiation are also irreversible.

2) Heat produced by the passage of a current through a resistance is irreversible. Because even when the current is reversed the same effect is observed.

3) Work done against friction is irreversible, because in reversing the direction of motion again work has to be done against friction.

4) Rusting of iron.

3.7 Second law of thermodynamics :

[Kelvin and Clausius statement]

Kelvin's statement :

"It is impossible to get continuous supply of work from a body by cooling it to a temperature lower than that of its surroundings."

In a heat engine the working substance extracts heat from the source, converts a part of it into work and rejects the rest to a sink at a lower temperature. The temperature of the source must be higher than the surroundings & engine will not work when the temperatures of source and sink are the same. No engine has ever been constructed which converts all the heat absorbed from the source into work without rejecting a part of it to the cold body. As the engine absorbs more and more heat from the hot body, the latter suffers a continuous fall in temperature and if continuous supply of work is desired, the hot body will in the long run become as cold as its surroundings.

Then no heat flow will be possible, the engine will stop working and hence no mechanical work will be obtained. Means we cannot obtain a continuous supply of work from a single supply of it, i.e. the presence of colder body is a must for the continuous conversion of heat into work.

Clausius statement :

"It is impossible for a self acting machine working in a cyclic process, unaided by external agency, to transfer heat from a body at a lower temperature to a body at a higher temperature." Or "Heat cannot flow of itself from a colder body to a hotter body."

This statement is applicable in case of refrigerator, a heat engine working in the backword direction. Means the natural flow of heat is always flow from a hot body to a cold body. If heat is transferred from cold body to hot body, work will have to be done by external agency. A refrigerator is a device which transfers heat from a colder body to hotter body by doing external work on the working substance.

Thus the second law of thermodynamics plays an important role for practical devices. Such as heat engine & refrigerators. **The first law gives only the relation between the work done and heat produced. But the second law gives the conditions under which heat can be converted into work.**

3.8   Heat - engines :

"It is a practical machine which converts heat into mechanical work is called a heat engine."

Heat engine absorbs heat at higher temperature, converts part of it into mechanical work, and rejects the remaining heat of a low temperature. In this process a working substance is used. In steam engines working substance is water vapour, and in all gas engines the working substance is a combustible mixture of gases.

The working substance goes through certain changes of pressure, volume and temperature, and then returns to the initial state. The complete changes through which the working substance undergoes from its initial state and back to its starting state constitute one cycle of operation.

Now, **"efficiency of heat engine is defined as the ratio of mechaical work done by the engine in one cycle to the heat absorbed from the high temperature source."**

Thus,

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

where, Q_1 = heat absorbed from the source at high temperature.

Q_2 = heat rejected to a sink at low temperature.

$(Q_1 - Q_2)$ = mechanical work done by the engine in one cycle.

3.9 : Carnot's ideal heat engine :

In 1824 the French engineer Sadi Carnot conceived a theoretical engine which is free from all defects of practical engines.

Heat engines are used to convert **heat into mechanical work**. It is an ideal heat engine & its efficiency is maximum. Carnot's heat engine requires the following requirements shown in fig.

1) Cylinder : A cylinder having perfectly non-conducting walls, a perfectly conducting base and is provided with a perfectly non-conducting piston which moves without friction in the cylinder. The cylinder contains one mole of perfect gas as the working substance.
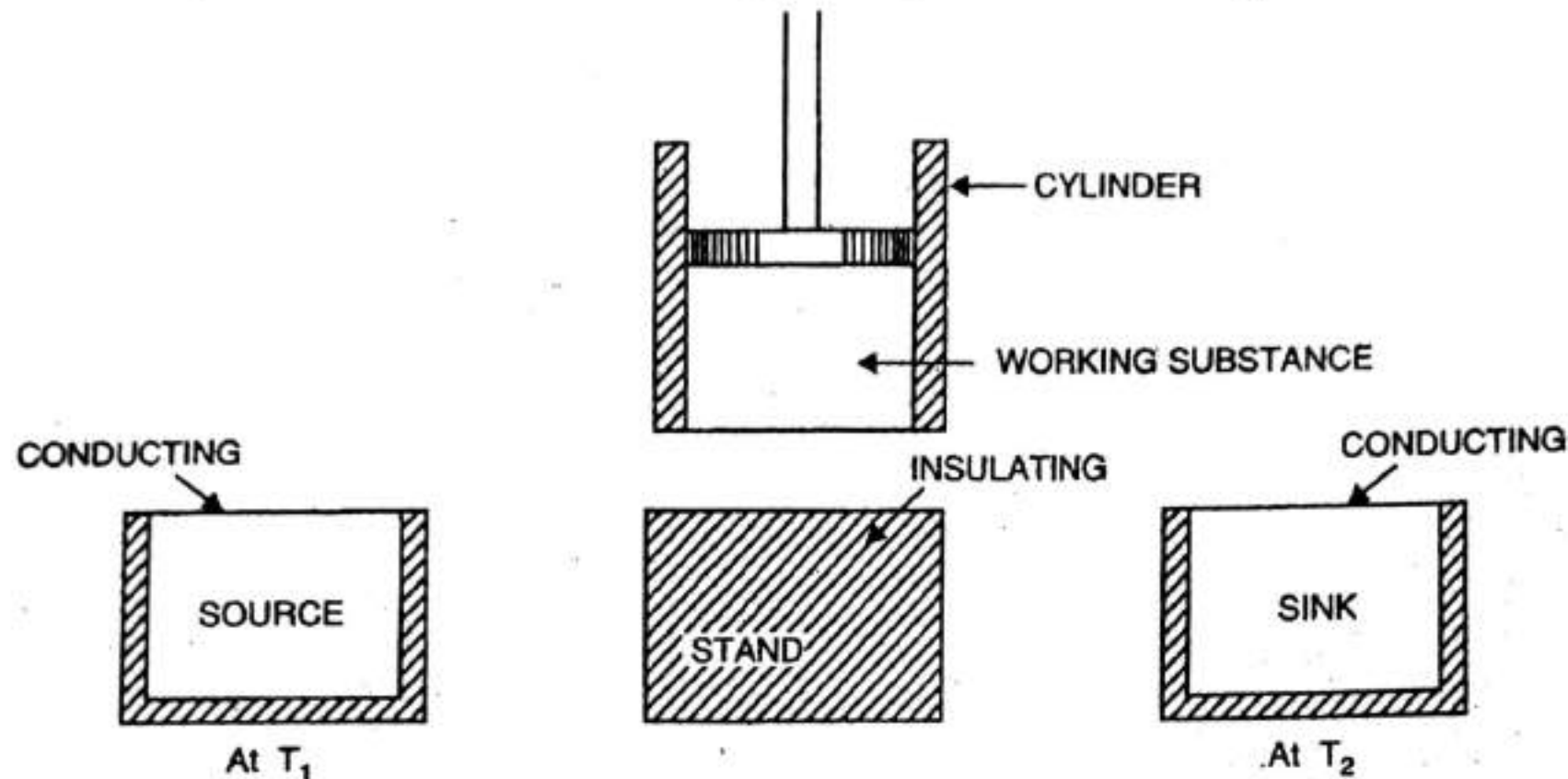
2) Source : A reservoir maintained at a constant temperature T_1 from which the engine can draw heat by perfect conduction. It has infinite thermal capacity and any amount of heat can be drawn from it at constant temperature.

3) Heat insulating stund : A perfectly non-conducting platform acts as a stand for adiabatic processes.

4) Sink : A reservoir maintained at a constant lower temperature T_2 $(T_2 < T_1)$ to which the heat engine can reject any amount of heat.

The thermal capacity of sink is infinite so that its temperature remains constant at T_2.

3.10 : Carnot's cycle :

"The process occuring in the engine are reversible. The reversible cyclic process consists of a sequence of isothermal and adiabatic curves on a P - V diagram is known as Carnot's cycle."

The state of a system is uniquely determined by a point on P - V diagram known as indicator diagram, each point representing an equillibrium state. A line joining a series of such points represents a succession of an infinitely large number of equillibrium states infinitesimally close to

each other and shows aquasi-static process. **Quasi static** process is a process carried out sufficiently slowly so that the system under consideration remains at all times arbitrarily close to equillibrium.

Work done in Carnot's cycle :

In below fig. of Carnot's cycle the working substance is supposed to undergo the following four quasi-static operations.
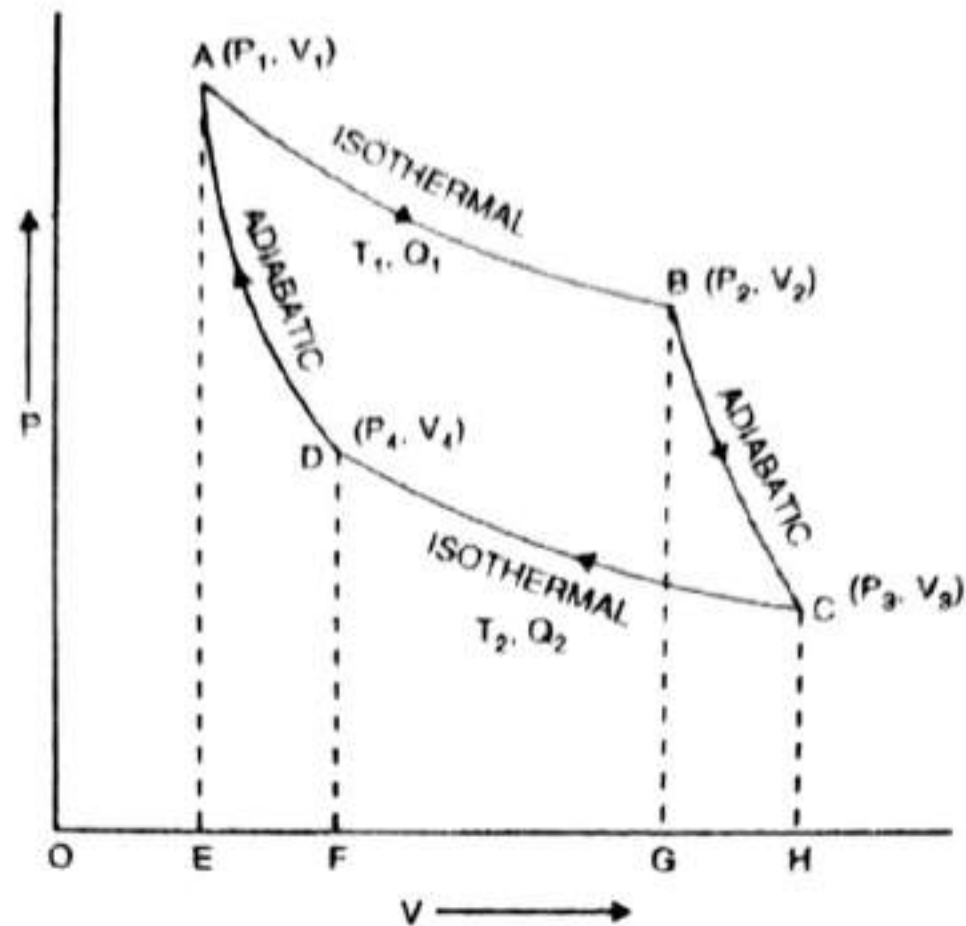
1) Isothermal expansion :

Suppose the cylinder contains one mole of perfect gas. Let the initial temperature of the gas in the cylinder be $T_1 k$, and temperature of heat source is also $T_1 k$. Initially the cylinder is placed on the source, so that the gas acquire the temperature T_1 and then allowed to undergo quasi-static expansion. As the gas expands its temperature tends to fall. Heat passes into the cylinder through the perfectly conducting base which is in contact with the source. Therefore the gas undergoes slow isothermal expansion at the constant temperature T_1.

Let the working substance during isothermal expansion goes from its initial state $A(P_1, V_1, T_1)$ to the state $B(P_2, V_2, T_2)$ at constant temperature T_1 along AB. The substance absorbs heat Q_1 from the source at T_1 and does work W_1 is given by,

$$Q_1 = W_1 = \int_{V_1}^{V_2} PdV = RT_1 \log_e \frac{V_2}{V_1}$$

$$\boxed{\therefore Q_1 = \text{area ABGEA}}$$

------ (1)

2) **Adiabatic expansion :**

Now the cylinder is removed from the source & is placed on the insulating stand. The gas is allowed to slow adiabatic expansion, performing external work at the expense of its internal energy, until its temperature falls to T_2.

Such operation is represented by the adiabatic BC, from the initial state $B(P_2, V_2, T_1)$ to the state $C(P_3, V_3, T_2)$. There is no transfer of heat the temperature of the substance falls to T_2 and it does some external work W_2.

i.e.

$$W_2 = \int_{V_2}^{V_3} P dV = K \int_{V_2}^{V_3} \frac{dV}{V^\gamma}$$

($\because$ During the adiabatic process $PV^\gamma = $ constant $= K$)

$$W_2 = \frac{KV_3^{1-\gamma} - KV_2^{1-\gamma}}{1-\gamma}$$

$$W_2 = \frac{P_3 V_3 - P_2 V_2}{1-\gamma} \qquad \left(\because P_2 V_2^\gamma = P_3 V_3^\gamma = K \right)$$

$$W_2 = \frac{RT_2 - RT_1}{1-\gamma}$$

$$\boxed{W_2 = \frac{R(T_1 - T_2)}{\gamma - 1} = \text{Area BCHGB}} \qquad \text{----- (2)}$$

3) Isothermal Compression :

The cylinder is now removed from the insultaing stand and is placed on the sink which is at temperature T_2. The piston is now very slowly moved inwards so that work is done on the gas. The temperature increases due to the heat produced by compression since the conducting base of the cylinder is in contact with the sink, the heat developed passes to the sink and the temperature of the gas remains constant at T_2. Thus the gas undergoes isothermal expansion at a constant temperature T_2 and gives up some heat to the sink.

Such operation shown in fig. by the isothermal CD, starting from state $C(P_3, V_3, T_2)$ to the state $D(P_4, V_4, T_2)$. In this process the substance rejects heat Q_2 to the sink at T_2 and work W_3 is done on the substance is given by

$$Q_2 = W_3 = \int_{V_3}^{V_4} P dV = RT_2 \log_e \frac{V_4}{V_3}$$

$$Q_2 = W_2 = -RT_2 \log_e \frac{V_3}{V_4}$$

$$\boxed{Q_2 = W_2 = \text{Area CHFDC}}$$

------ (3)

(-ve sign indicates that work is done on the working substance.

4) Adiabatic compression :

Now the cylinder is removed from the sink and again placed on the insulating stand. The piston is slowly moved inwards so that the gas is adiabatically compressed and the temperature rises. The adiabatic compression is continued till the gas comes back to its original condition i.e. state $A(P_1, V_1, T_1)$. The operation is represented by adiabatic DA, starting from $D(P_4, V_4, T_2)$ to the final state $A(P_1, V_1, T_1)$ in this process, work W_4 is done on the substance i.e.

$$W_4 = \int_{V_4}^{V_1} PdV$$

$$W_4 = -\frac{R(T_1 - T_2)}{\gamma - 1}$$

$$\boxed{W_4 = \text{Area DFEAD}} \qquad\qquad \text{------ (4)}$$

Work done by the engine per cycle :

During the above cycle the working substance absorbs an amount of heat Q_1 from the source & reject Q_2 to the sink. Therefore, the net amount of heat absorbed by the gas per cycle.

$$= Q_1 - Q_2$$

The net work done by the engine per cycle $\qquad = W_1 + W_2 + W_3 + W_4$

$$(\because W_2 = -W_4)$$

$$= W_1 + W_3$$

From the graph, the net work done per cycle =

area ABGEA + area BCHGB - area CHFDC - area DFEAD   ------ (5)

Thus, the area enclosed by Carnot's cycle consisting of two isothermals and two adiabatics gives the net amount of work done per cycle.

In this process,

Net heat absorbed = Net work done per cycle.

i.e. $Q_1 - Q_2 = W_1 + W_3$

$$Q_1 - Q_2 = RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_3}{V_4}$$ ----- (6)

Since the points A and D lie on the same adiabatic DA.

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$ ------ (7)

Similarly, Point B and C lie on the same adiabatic BC,

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

From eqn (7) & (8) we have,

$$\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

Or $\quad \dfrac{V_1}{V_4} = \dfrac{V_2}{V_3}$

Or $\quad \dfrac{V_2}{V_1} = \dfrac{V_3}{V_4}$

But net work done $= Q_1 - Q_2$

$$W = RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_2}{V_1}$$

$$\therefore \quad W = (Q_1 - Q_2)$$

$$W = R(T_1 - T_2) \log_e \frac{V_2}{V_1}$$

------- (9)

Efficiency :

"It is rate of quantity of heat converted into work (useful output) per cycle to the total amount of heat absorbed per cycle."

Therefore efficiency $\eta = \dfrac{\text{useful output}}{\text{Input}}$

$$\eta = \frac{W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = \frac{R(T_1 - T_2) \log_e \dfrac{V_2}{V_1}}{RT_1 \log_e \dfrac{V_2}{V_1}}$$

$$\eta = \frac{T_1 - T_2}{T_1} \quad \text{------- (10)}$$

Or $$\eta = 1 - \frac{T_2}{T_1} \quad \text{------- (11)}$$

Also we can write,

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Or $$1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Or $$\boxed{\frac{Q_2}{Q_1} = \frac{T_2}{T_1}} \quad \text{------ (12)}$$

From eqn (11) efficiency depends only upon the temperature of the source and sink and is less than unity,

Also eqn (12) shows that the ratio of the heat absorbed from the source to the heat rejected to the sink is equal to the ratio of absolute temperature of the source and the sink.

:: Multiple Choice Questions ::

1) In carnots cycle, the first step is :
   a) Isothermal expansion
   b) Isothermal compression
   c) Adiabatic expansion
   d) Adiabatic compression

2) The ratio of adiabatic and isothermal elasticities of a gas is :
   a) γ b) γ^2 c) γ^{-1} d) $\left(\sqrt{\pi}\sqrt{\gamma}\right)$

3) An engine works between the temperature 30^0k and 300^0k what is its efficiency
   a) 50 %   b) 47%   **c) 90%**   d) 10%

4) The efficiency of Carnots engine working between 127^0c and 27^0 c is :
   a) 25%   b) 50%   c) 75%   d) 100%

5) An adiabatic process occurs at constant
   a) Temperature   b) pressure   **c) heat**   d) none of these

6) An isothermal process occurs at constant.
   a) Temperature   b) pressure   c) heat   d) volume.

7) The efficiency of a reversible Carnots engine working between temperature T_1 & T_2 $(T_1 > T_2)$ is

a) $\dfrac{T_2}{T_1}$ b) $\dfrac{T_1}{T_2}$ c) $\left(1-\dfrac{T_2}{T_1}\right)$ d) $\left(\dfrac{T_1}{T_2}-1\right)$

8) The gas law $\dfrac{PV}{T}$ = constant is true for

a) Isothermal change
b) Adiabatic change
c) By the isothermal and adiabatic change
d) None of this

9) The work done during an adiabatic expresion of an ideal gas is :

a) $W = \dfrac{1}{1-\gamma}(P_1 V_1 - P_2 V_2)$

b) $W = \dfrac{1}{\gamma-1}(P_1 V_1 - P_2 V_2)$

c) $W = R(T_2 - T_1)$

d) $W = RT_1 \log e \dfrac{V_2}{V_1}$

10) A reversible heat engine can be 100% efficient, if the temperature of the sink is :
a) less than that of source
b) equal than that of source
c) 0°C
d) 0°k

11) Heat engines is practical machine which converts :
a) heat into mechanical work
b) heat into energy
c) mechanical work into heat
d) none of these.

12) Quasi – static process is a process carried out :

   a) **Very very slowly**
   b) Very fast
   c) Very fast to slowly
   d) None of these.

So that the system under consideration remains at all times arbitrarily close to the equilibrium state.

13) The door of running refrigerator inside a room is left open. Mark the corrects statement.
   a) The room will be cooled slightly.
   b) **The room will be warmed up gradually.**
   c) The temperature of the room will remain unaffected.
   d) The room will be cooled to the temperature inside the refrigerator.

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Minimum Three Slides Daily

THANK YOU