T.Y.B.SC. PHYSICS

Fifth Semester Paper XV

Classical & Quantum Mechanics

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CHAPTER - 2

Origin of Quantum Mechanics

2.1 Introduction :

Before 1900, most of the phenomenon could be explained on the basis of classical Physics which is based on Newtons laws i.e. law of inertia, law of force and law of action and reaction.

The equations based on these laws are simple and can be explain successfully the motion of the object which are either directly observable or can made observable by simple instruments like microscope, in this way classical mechanics can be explained successfully the motion of bodies moving with non relativistic speed. With the discovery of electron exploration of microscopic system were started, but classical concepts can not be directly applied to the motion of electron in an atom or atomic phenomenon, which are not observable with the help of the instruments.

In 1900 Max Plancks introduce new concepts that absorption or emission of electromagnetic radiations takes placeas discrete quanta each of having energy $E = h\upsilon$ where υ is frequancy of radiation and h is the planks constant. These concepts led to a new mechanics which is known as quantum Mechanics. Also the limitation in the enery distribution in the spectrum of a black body was successfully explained by Max Plancks in 1900. This revolutionary concepts is origin of quantum theory.

2.2 Failure of classical Mechanics:

There are many exprimental results which could not explained on the basis of classical mechnics such as,

1. Stability of atom:

The electron revolving round the nucleus have centripetal accelration. According to classical theory every accelerated charged particle radiates energy in the form of electromagnetic waves. Hence orbiting electron being an accelerated charged particle, radiates energy continuously. Due to this continuous loss of energy of the electron, the radii of their orbits should fall into the nuleus. Thus the atom can not remain stable. Thus classical Physics failed to explain the stability of an atom.

2. The spectrual distribution of heat radiation from black body or black bodies. According to classical Physics energy changes of radiation takes place continuously. The quantity and quantity of the emitted radiation depends only on the temperature of the body and not on the nature of the body. More ever the radiation emitted at a particular temperature consist of wide range of frequancies with different intensities.

Classical Physics could not explain black body spectrum which assume the energy changes of radiation takes place continuously. For different wavelength Wein obtained thermodynamically the following relations

$$\lambda_m T = Const. \quad \text{and}$$

$$E_m \propto T^5 \quad E_m T^{-5} = Const.$$

First relation shows the wavelength corresponding to maximum emitted energy decrease in temperature and second relation shows direct variation of emissive power with fifth power of absolute temperature

By using (1) and (2)

$$\therefore \; E_\lambda d\lambda = \frac{A}{\lambda^5} f(\lambda T)\, d\lambda$$

Above relation good in shorter wavelength region but for longer wavelength it gives value of E_λ lower than the exprimental values. Also the energy distribution in thermal spectrum according to rayleigh's

$$E_\lambda = \frac{8\pi kT}{\lambda^4}$$

This is good for longer wavelength

Thus Weins and Rayleigh Jeans law do not agree with the experimental results. In Rayleigh Jeans, the energy increases without limit as wavelength decreases.

$$E_\lambda \rightarrow \infty \; as \; \lambda \rightarrow 0$$

Such consequanses is called as ultraviolet cotastrophe. hence classical physics fails to explain black body spectrum.

values of a free (conduction) electron near the surface of a metallic conductor is as shown in fig.

Let the free electron inside the metal have energies ranging from 0 to a maximum value E_m. Then the energy required to remove the electron from the energy level E_s, will be the photo-electric work function ω_0 given by

$$\omega_0 = h\nu_0 = (E_s - E_m)$$

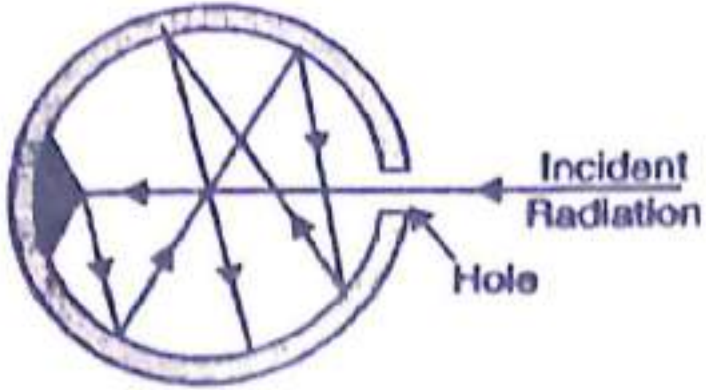
where ν_0 is the threshold frequency.

Now, consider an electron having an energy E_i less E_m than . If a photon of energy incident on the surface of the metallic conductor ejects this electron from the metal it will have a kinetic energy given by

$$E_{k(\max)} = \frac{1}{2}mv_{\max}^2 = h\nu - (E_s - E_i)$$

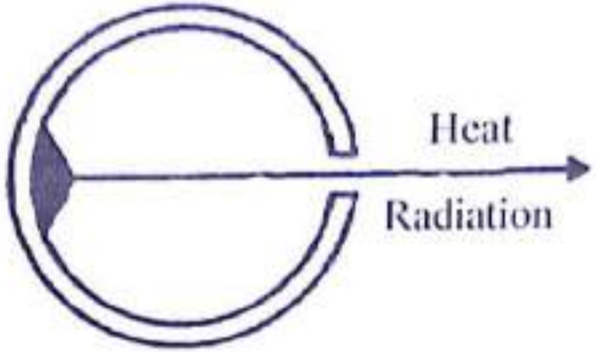
But E_i can have any value between 0 and E_m. Therefore, the kinetic energies and velocities of the electrons ejected by the action of photon of frequency ν will vary over a wide range. The maximum kinetic energy is given by

$$E_{k(\max)} = \frac{1}{2}mv_{\max}^2 = h\nu - (E_s - E_m) = (h\nu - E_s) + E_m$$



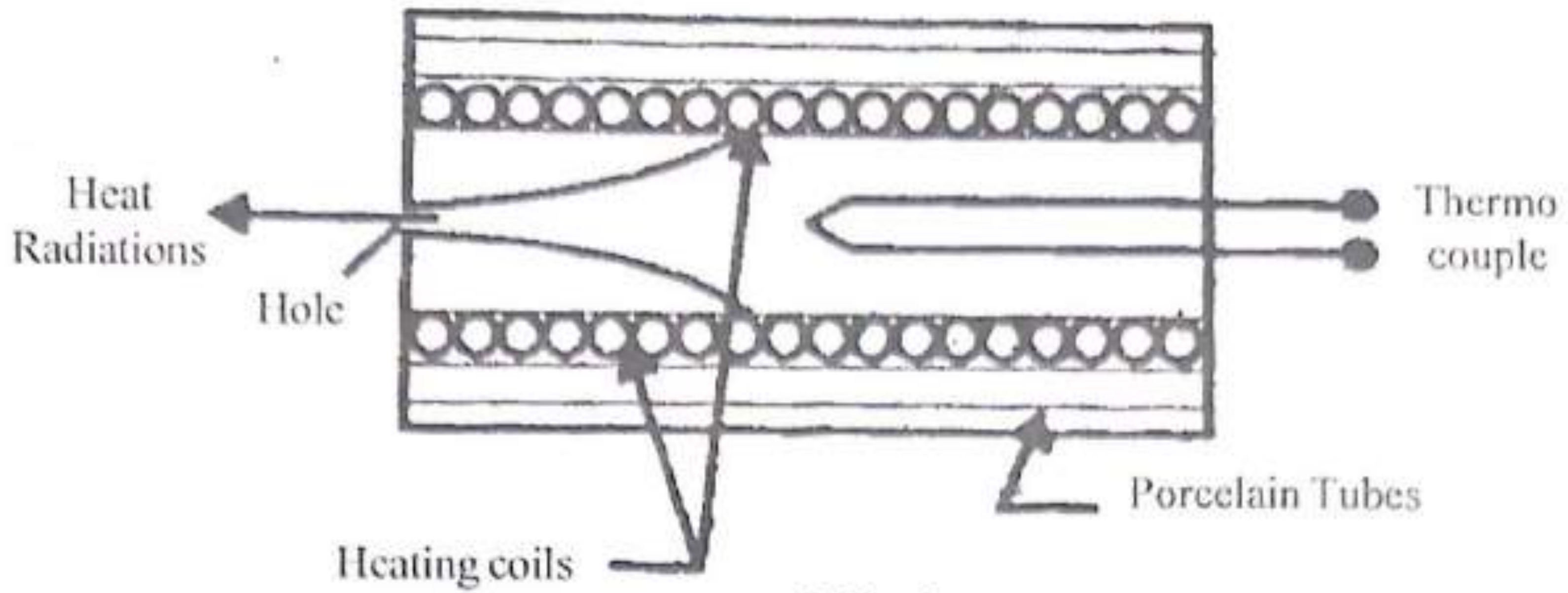
[Fig. Black body as a absorbor]

As shown in fig. the inner surface of a hollow copper sphere is coated with lamp black. A fine hole is made and conical projection is made just in front of the hole. When radiations enter the hole, they suffer multiple reflections and are completely absorbed. This body acts as a black body absorber. When this body is placed in a bath at a fixed temperature, the heat radiations are given out of the hole (fig.) A hole acts as a black body radiator.



[Fig. radiator]

Another type of black body due to Wien is shown in fig. It consists of long metallic tube blackened inside and surrounded by concentric porcelain tubes. The tube is heated by an electric current flowing in a wire wound around it. The radiation passes through a number of limiting diaphragms & issues through a narrow opening in the wall of the tube. The temperature of the central part of the interior is measured by thermocouple arrangement.

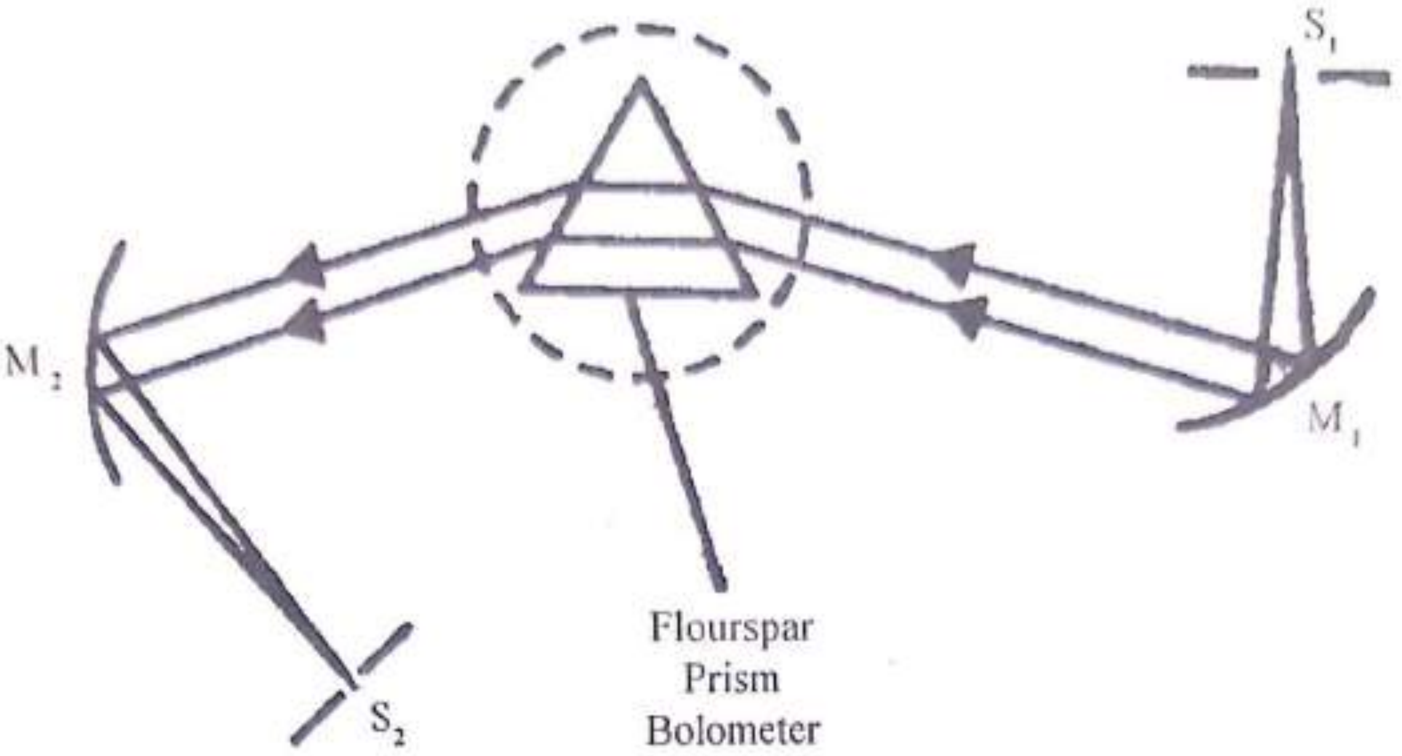


[Fig.]

Thus A blackbody, is one which absorbs all the radiations incident on it and emits, at any given temperature, a characteristic spectrum. This spectrum contains all frequencies i.e. is continuous) but the intensity associated with these frequencies differs in a manner which was the subject of extensive experimental investigations. This spectral distribution of intensities, while varying with the temperature of the emitting black body proved to be independent of its material composition. (Kirchoff's law)

Distribution of Energy:

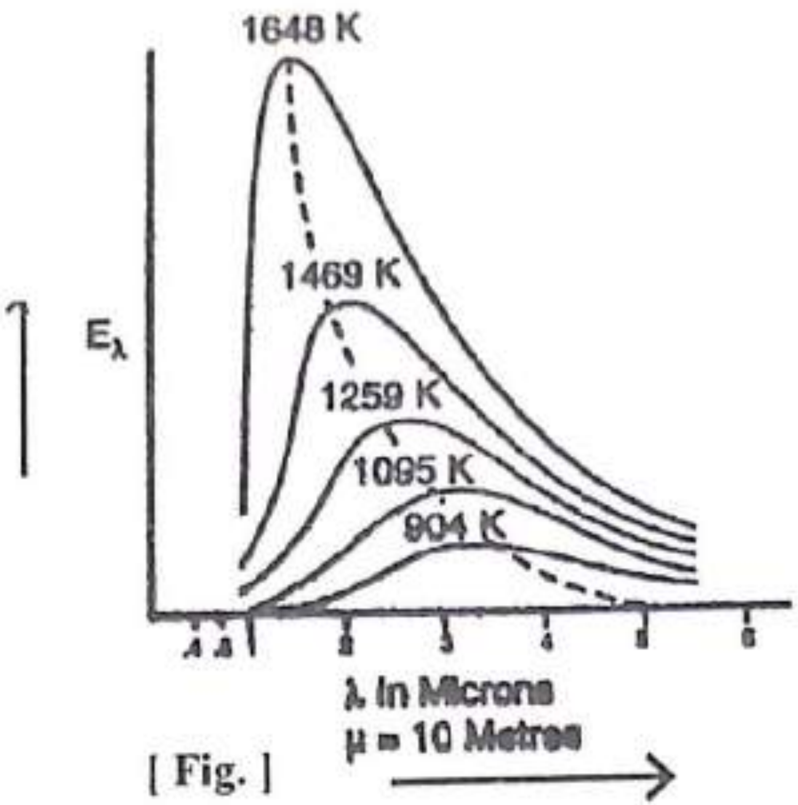
Lummer and Pringsheim in 1899 made experiments to determine the distribution of energy among the radiations of different wavelengths emitted by a blackbody at various temperatures.



[Fig.

The experimental arrangement consists of a carbon tube heated electrically (fig). The radiations from this tube are allowed to be incident on concave mirror M_1 through a slit S_1. The parallel reflected beam is disperesed by a Fluorspar prism. The dispersed beam is focused on a line bolometer with the help of the mirror m_2. The intensity of radiations corresponding to different wavelengths is measured with the help of Bolometer. The deflection in the galvanometer (of bolometer) is proportional to the energy of the heat radiations.

To find the energy distribution for different wavelengths the fluorspar prism is gradually rotated. The Results obtained by Lummer and Pringsheim are shown in fig. Each curve represents the intensity of radiation E_λ with wavelength for a given temperature of the source.



[Fig.]

Results :

1) At a given temperature, the energy is not uniformly distributed in the radiation spectrum of a hot body.

2) At a given temperature the intensity of radiation increases with increase in wavelength and at a particular wavelength its value is maximum. With further increase in wavelength the intensity of heat radiations decreases.

3) With increase in temperature λ_m decreases. λ_m is the wavelength at which maximum emission of energy takes place. The points on the dotted line represent λ_m at various temperatures.

4) For all wavelengths, an increase in temperature causes an increases in energy emission.

5) The area under each curve represents the total energy emitted for the complete spectrum of a particular temperature. This area increases with increase in the temperature of the body. This area is directly proportional to the fourth power of the temperature of the body.

i.e. $E \propto T^4$ ----- (1)

This represents Stefan Boltzmann's law.

2.4 Planck's Quantum Theory :-Planck's quantum postulates:

This failure of classical mechanics led Planck (in 1900) to the discovery that radiation is emitted in quanta whose energy is $E = h\nu$. In 1901, Planck was able to derive an empirical formula to explain the experimentally observed distribution of energy in the spectrum of a black body, on the basis of his revolutionary hypothesis known as quantum theory of heat radiation. According to this theory, the energy distribution is given by

$$E_\lambda d_\lambda = \frac{8\pi hc}{\lambda^5\left[e^{hc/\lambda kT} - 1\right]} d\lambda$$

This relation agrees (and hence completely fit) with the experimental curves obtained in Lummer and Pringesheim experiment.

This formula of distribution of energy with wavelengths, on the basis of his quantum concept, was deduced using following assumptions, which may be called as Planck's quantum postulates. These are :

Planck's quantum postulates :

1. A black body contains the atomic oscillators capable of vibrating with all possible frequencies. An oscillator of frequency v can not have any energy but only discrete values of energy given by

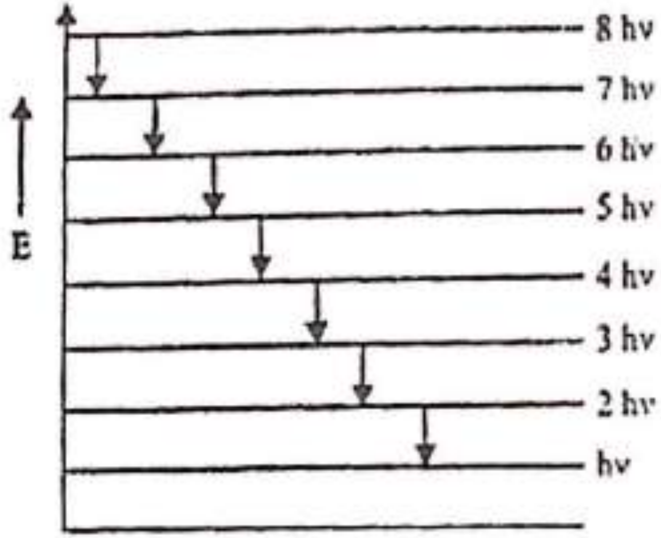
$$E_n = nh\nu$$

where $n = 0, 1, 2, 3....$

v = oscillator frequency

h = Planck's constant.

2. The atomic oscillator can not emit or absorb energy continuously. But it can emit or absorb the energy in discrete units of energy, a tiny packet called 'quanta'. The quantum energy is due to jump of the oscillator from one energy level to the other and is given by



[Fig: Energy states of atom.]

$$\nabla E_n = \nabla n(hv)$$

where ∇n = difference of levels. For example, if the transition energy levels are n = 8 and n = 5, then

$$\nabla E = E_8 - E_5 = 8hv - 5hv = 3hv$$

From the first assumption, given by Eq. It is clear that each quantum of energy depends upon the frequency of radiation (v) and is not divisible. Moreover, the exchange of energy between matter and radiation is in terms of integral multiple of hv. Therefore, the total energy exchanged may be $0, hv, 2hv, 3hv, ...nhv$ in discrete manner. However, according to classical theory, the atom can possess any amount of energy from zero and infinity, hence exchange are energies are continuous i.e. from 0 to which is inadequate to quantum concept.

According to Planck's theory, the atom is associated with the allowed energy states called quantum states and its energies are quantized as shown in fig. The atom can emit or absorb the radiation i.e. quantum of energy by jumping from one energy state to the other.

The oscillator emits energy only when it passes from a higher energy state to a lower energy state and absorbs energy when it goes from a lower energy state to a higher energy state. No emission or absorption of energy takes place when the oscillator is in a given state. The smallest amount of energy which can be emitted or absorbed by the oscillator is hv. In other words a radiation of frequency v is emitted as quantum of energy $E = h\nu$ where h is Planck's universal constant having a value equal to $6.62\times10^{-27}\, erg.\sec$ (or $6.62\times10^{-34}\, Joule$ sec). The quantum is the basic unit of energy and cannot be subdivided. It is the quantum of energy and Einstein named it as a photon.

This theory has been able to explain a number of phenomena which could not be explained according to the classical conception of radiation. For example, it gives a very satisfactory explanation of variation of specific heat of solids with temperature, photo-electric effect, Compton effect etc. and has been successfully applied by Bohr in the theory of the hydrogen spectrum.

2.5 Linear momentum of photon in terms of wave vector :

Photon : A photon is a particle of zero rest mass. Its dynamical variables are energy and momentum It has zero charge and spin equal to one quantum unit (i.e. $\frac{h}{2\pi} = \hbar$)

Energy $E = h\upsilon$ momentum $P = \frac{h}{\lambda}$

If is wavelength of light then frequancy

$$\upsilon = \frac{c}{\lambda}$$, c is velocity of light.

Each quantum of energy of this radiation is known as photon

Now energy of photon, $E = h\upsilon = \frac{hc}{\lambda}$

Also according to Einstain $E = mc^2$

or $m = \frac{E}{c^2}$

Mass of Photon of energy $h\upsilon$ is $h\upsilon = \frac{h\upsilon}{c^2}$

Becouse theory of relativity shows a mass m has an equivalent to

Now, Momentum(P)=Mass $\times V$elocity

$$P = \frac{h\upsilon}{c^2}.c = \frac{h\upsilon}{c} = \frac{h}{\lambda} \qquad \text{------ (1)}$$

Two charges or two magnetic poles exerts a force on each other due to exchange of photons, it interacts with all the charged particles and some nutral particles. Also a photon bahaves as a small elastic sphere for all purposes. When a photon collides with an electron both momentum and energy are conserved. When collision between two elastic spheres also shows energy and momentum is conserved, i.e. varified by compton using x-rays experiment.

Let X-ray of frequancy υ collide with an electron of mass m then,

$$h\upsilon = \frac{1}{2}m\upsilon^2 + h\upsilon' \qquad \text{------ (2)}$$

here υ is velocity of electron

υ' is frequancy of new radiation

The magnitude of the linear momentum of a photon of wavelength λ is given by

$$P = \frac{h}{\lambda}$$

Divide and multiply by 2π we have,

$$P = \frac{h}{2\pi}.\frac{2\pi}{\lambda} = \frac{h}{2\pi}.|\vec{K}| \qquad \text{------ (3)}$$

here $|\vec{K}| = \frac{2\pi}{\lambda}$ is magnitude of wave vector $\vec{K}$. But we know $\hbar = \frac{h}{2\pi}$ then (3) is

$$P = \hbar|\vec{K}| \qquad \text{------- (4)}$$

which shows linear momentum of photon in terms of wave vector $\vec{K}$ Also photon energy in terms of angular velocity

We know

$$E = \frac{hc}{\lambda} = h\upsilon \quad Where\ \upsilon = \frac{c}{\lambda}$$

Divide and multiply by 2π we have,

$$E = \frac{h}{2\pi}.2\pi\upsilon = \frac{h}{2\pi}\omega \quad Here\ 2\pi\upsilon = \omega \ \ and\ \frac{h}{2\pi} = \hbar$$

$$E = \hbar\omega \qquad \text{-------} \ (5)$$

2.6 Planck's Radiation Law :

Planck's modified Rayleigh-Jeans law with his assumption that a black body is associated with large number of harmonic oscillators. The oscillators have energies.

$$E = nh\nu,\ n = 0,1,2,... \qquad \text{-------} \ (1)$$

According to Boltzmann distribution law, the number of oscillators with energies,

$0, h\nu, 2h\nu, 3h\nu, ...$ would be $N_0, N_e e^{-h\nu/kT}, N_0, e^{-2h\nu/kT} ...$ respectively.

Thus, the total number of oscillators is given by,

$$N = N_0 + N_0 e^{-h\nu/kT} + N_0 e^{-2h\nu/kT} + ... + N_0 e^{-nh\nu/kT} \qquad \text{-------} \ (2)$$

when $\quad N_0 =$ the number of oscillators having zero energy

$k =$ Boltzmann constant.

Putting $\quad \frac{h\nu}{kT} = \alpha$ in Eqn. (ii), we get

$$N = N_0 + N_0 e^{-\alpha} + N_0 e^{-2\alpha} + N_0 e^{-3\alpha} + ... + N_0 e^{-n\alpha}$$

$$= N_0 \left(1 + e^{-\alpha} + e^{-2\alpha} + ... + e^{-n\alpha}\right)$$

$$= \frac{N_0}{\left(1 - e^{-\alpha}\right)}$$

or $\quad \frac{N_0}{N} = \left(1 - e^{-\alpha}\right) \qquad \text{-------} \ (3)$

The total energy of all the oscillators is,

$$E = 0 + h\nu N_0 e^{-a} + 2h\nu N_0 e^{-2a} + 3h\nu N_0 e^{-3a} + \ldots + nh\nu e^{-na}$$

$$E = h\nu N_0 e^{-a}\left(1 + 2e^{-a} + 3e^{-2a} + \ldots + ne^{-(n-1)a}\right) \qquad \text{-------} \ (4)$$

and $$Ee^{-a} = h\nu N_0 e^{-a}\left(e^{-a} + 2e^{-2a} + 2e^{-3a} + \ldots + ne^{-na}\right) \qquad \text{-------} \ (5)$$

Subtracting Eqn. (v) from (iv), we have,

$$E\left(1 - e^{-a}\right) = h\nu N_0 e^{-a}\left(1 + e^{-a} + e^{-2a} + e^{-3a} + \ldots\right)$$

$$= \frac{h\nu N_0 e^{-a}}{1 - e^{-a}}$$

$$E = \frac{h\nu N_0 e^{-a}}{\left(1 - e^{-a}\right)^2} \qquad \text{-------} \ (6)$$

The average energy per oscillator is thus,

$$E = \frac{E}{N} = \frac{N_0}{N} \cdot \frac{h\nu e^{-a}}{\left(1 - e^{-a}\right)^2}$$

From Eqn. (iii), we have

$$\overline{E} = \left(1 - e^{-\alpha}\right) \frac{h\nu e^{-\alpha}}{\left(1 - e^{-\alpha}\right)^2} = \frac{h\nu e^{-\alpha}}{\left(1 - e^{-\alpha}\right)}$$

or $$\overline{E} = \frac{h\nu}{e^{-\alpha} - 1} = \frac{h\nu}{e^{h\nu/kT} - 1} \qquad \text{------- (7)}$$

The number of oscillators per unit volume in the frequency range v and v + dv is

$$N_\nu = \frac{8\pi \nu^2}{c^3} . d\nu$$

Therefore, the energy density i.e., the total energy per unit volume in the frequency range v and v + dv is product of $\overline{E}$ and N_ν,

$$E_\nu d\nu = N\nu . \overline{E} = \frac{8\pi \nu^2}{c^3} . d\nu . \frac{h\nu}{e^{\alpha} - 1}$$

or $$E_\nu d\nu = \frac{8\pi h \nu^3}{c^3} \left[\frac{1}{e^{h\nu/kT} - 1} \right] d\nu \qquad \text{------- (8)}$$

Eq^n.(8) is called Planck's radiation law. This law can explain all the experimental results for black body radiation.

The energy density $E_\nu d\nu$ can be written in terms of the wavelength range λ and $\lambda + d\lambda$ from

$$\nu = \frac{c}{\lambda}$$

$$|d\nu| = \left|-\frac{c}{\lambda^2} d\lambda\right| \qquad \text{or} \qquad d\nu = \frac{c}{\lambda^2} d\lambda$$

$$\therefore \; E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{hc/\lambda kT} - 1}\right] d\lambda \qquad \text{------- (10)}$$

Wein's law and Rayleigh-Jeans law can be shown as approximations of Planck's law as follows:

1. Wein's law :

At shorter wavelengths or larger frequencies, $\frac{hc}{\lambda kT} >> 1$ i.e. $\exp(hc/\lambda kT) >> 1$ therefore neglecting 1 in the denominator of Eq^n.

$$E_\lambda d\nu = \frac{8\pi hc}{\lambda^5} . e^{hc/\lambda kT} d\lambda$$

This is Wein's law of radiation.

2. **Rayleigh-Jeans law :**

At longer wavelengths, $\frac{hc}{\lambda kT} << 1$, expanding

$$\exp(hc/\lambda kT) = 1 + \frac{hc}{\lambda kT} + \frac{1}{2!}\left(\frac{hc}{\lambda kT}\right)^2 + \ldots$$

as $\frac{hc}{\lambda kT} << 1$, neglecting higher power terms, we get

$$\exp(hc/\lambda kT) \approx 1 + \frac{hc}{\lambda kT}$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}.\frac{1}{hc/\lambda kT}d\lambda = \frac{8\pi}{\lambda^4}.kTd\lambda \qquad \text{------ (11)}$$

Eq^n. becomes

Eq^n.(11) is Rayleigh-Jeans law.

Thus Planck's law of radiation is applicable to all the wavelenghs in the black body spectrum.

2.7 Einstein's Equation : Quantum Theory of Photoelectric Effect :

In 1905, Einstein was successful in explaining the photoelectric effect and the laws of photo-electric effect making use of quantum theory of radiation. He assumed that :

Assumptions :

(i) A light of frequency ν consists of quanta of energy or photons. Each photon has an energy $h\nu$ and travels with the velocity of light.

(ii) In photo electric effect one photon is completely absorbed by one electron in the photo-cathode.

Derivation of Equation : Consider a photon of energy $h\nu$ which ejects a photo-electron from a material, with a velocity u, then

$$h\nu = \frac{1}{2}mv^2 + \omega \qquad \text{------- (1)}$$

where ω is the work done or energy spent in just bringing the electron outside the surface. Thus a part of the energy of the incident photon is used by the electron to come out of the metal surface and the remainder is stored in it as kinetic energy. The fastest electron is, therefore, produced nearest to the surface because it requires the least energy to be 'unchained' from this surface.

If ω_0 is the **photo-electric work function**. i.e. the energy required to eject an electron just out of the surface then the frequency of light ν_0 required for the purpose is given by

$$\omega_0 = h\nu_0$$

The frequency ν_0 is known as **threshold frequency.**

It is that frequency of light which when falling on a photo sensitive surface is just able to liberate electrons without giving them any additional energy.

It is clear that a frequency less that ν_0 will not be able to produce the photo-electric effect as an electron can absorb only one photon and no more. The threshold frequency ν_0 is thus given by

$$\nu_0 = \frac{\omega_0}{h}$$

When a radiation of frequency v greater than the **threshold frequency** is incident on the substance the maximum velocity v_{max} with which the electron is ejected is given by the relation.

$$h\nu = \frac{1}{2}mv_{max}^2 + \omega_0 \qquad \text{-------} \ (2)$$

which is known as **Einstein's photo-electric equation.**

Now $\omega_0 = h\nu_0$

Hence the above equation can be put in the form

$$h\nu = \frac{1}{2}mv_{max}^2 + h\nu_0$$

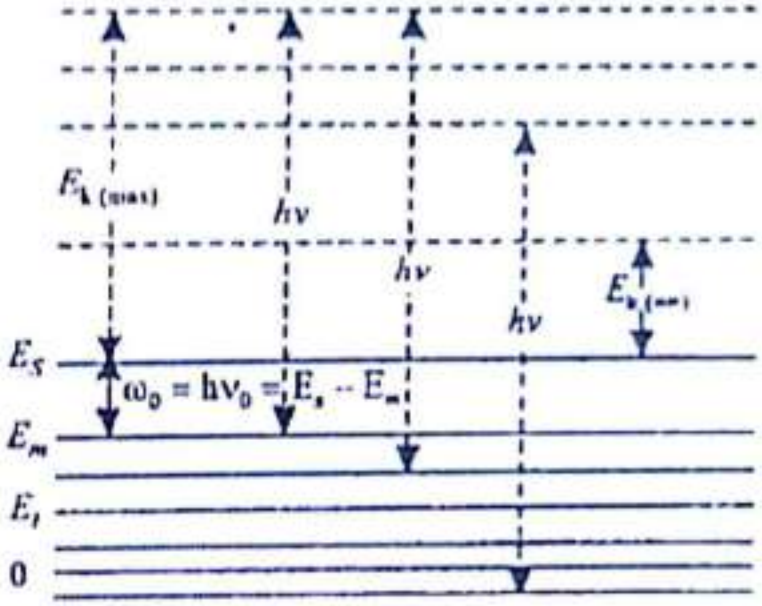
or $$\frac{1}{2}mv_{max}^2 = h(\nu - \nu_0)$$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \qquad \text{------- (3)}$$

where λ and λ_0 are wavelengths corresponding to frequencies ν and ν_0.

Thus, an increase in the frequency of the incident light increases the velocity with which a photo-electron is ejected.

Why all the photo-electrons do not have same energy? In the case of surface photo-electric effect in metallic conductors, it is generally assumed that ω_0 is the work done on the electron in bringing it out of the surface of the metal where there exists a potential barrier. But according to the theory of electron conduction in metals the conduction electrons have a wide range of energy inside the metal. The possible energy



[Fig.]

values of a free (conduction) electron near the surface of a metallic conductor is as shown in fig.

Let the free electron inside the metal have energies ranging from 0 to a maximum value E_m. Then the energy required to remove the electron from the energy level E_s, will be the photo-electric work function ω_0 given by

$$\omega_0 = h\nu_0 = \left(E_s - E_m\right)$$

where ν_0 is the threshold frequency.

Now, consider an electron having an energy E_i less E_m than . If a photon of energy incident on the surface of the metallic conductor ejects this electron from the metal it will have a kinetic energy given by

$$E_{k(\max)} = \frac{1}{2} m v_{\max}^2 = h\nu - \left(E_s - E_i\right)$$

But E_i can have any value between 0 and E_m. Therefore, the kinetic energies and velocities of the electrons ejected by the action of photon of frequency ν will vary over a wide range. The maximum kinetic energy is given by

$$E_{k(\max)} = \frac{1}{2} m v_{\max}^2 = h\nu - \left(E_s - E_m\right) = \left(h\nu - E_s\right) + E_m$$

The minimum kinetic energy is given by

$$E_{k(\min)} = \frac{1}{2} m v_{\min}^2 = h\nu - (E_s - 0) = (h\nu - E_s)$$

Thus, when a monochromatic beam of light is incident on the surface of a metal, the photo-electrons emitted will not all of them, have the same kinetic energy and velocity. The kinetic energy of the emitted electron will vary between $E_{k(\max)}$ and $E_{k(\min)}$. It is for this reason that when a potential difference is applied in the opposite direction the photo-electric current reduces to zero gradually and not abruptly as the opposing potential difference is increased and becomes zero only when V is given by

$$Ve = \frac{1}{2} m v^2 = h\nu - \omega_0 = h\nu - h\nu_0$$

Explanation of the laws of photo-electric emission.

(i) It is clear from Einstein's photo-electric equation.

$$\frac{1}{2} m v^2 = h(\nu - \nu_0)$$

that if the frequency of the incident photon $\nu < \nu_0$, the threshold frequency the kinetic energy of the emitted electron is negative, which is meaningless.

Hence no-photo electrons are emitted if the frequency of incident light is less than the threshold frequency.

(ii) For a frequency $\nu > \nu_0$ the kinetic energy of the photo-electron is directly proportional to $(\nu - \nu_0)$. As is a constant the kinetic energy of the photo-electron increases with increase in frequency of the incident light. Thus the maximum velocity of the photo-electron depends upon the frequency of incident light and is independent of intensity of light.

(iii) Since photo-electrons are absorbed in single units, there is a localisation of energy. The photons are, therefore, immediately absorbed and there is no significant time delay in the emission of photo-electrons.

(iv) If the intensity of light is increased keeping frequency same, then more photons are incident on the metal surface each photon having the same energy. Hence more electrons are ejected. As an electron can absorb only one photon each electron will have the same maximum energy and will be ejected with the same maximum velocity. Hence an increase in the intensity of incident light only increases the number of photo-electrons ejected and not their velocity.

(v) The stopping potential V is given by

$$Ve = h\nu - h\nu_0$$

As $h\nu_0$ is a constant, the stopping potential is proportional to the frequency of the incident photon. V is independent of intensity.

(vi) In Einstein's photo-electric equation $\frac{1}{2}mv^2 = h\nu - h\nu_0$ no factor depends upon temperature. Hence the rate of emission of photo-electrons is independent of temperature.

2.8 Compton effect :

Prof. A. H. Compton discoverd that, when x Ray of a sharply defined frequency were incident on a material of low atomic number like carban, they suffered a charge of frequency anscattering. The scattered beam contains two wavelength, In addition to the expected incident wavelength, there exsist a line of longer wavelength. The change of wavelength is due to loss of energy of incident x ray. This elestic interaction is known as comption effect.

In such cases an electron is also ejected with an energy depending upon its direction such electron is known as compton recoil electron.

There are following assumtions,

1) The X-ray of frequancy υ consist of photon of energy and momentum $\frac{h}{\lambda} = \frac{h\upsilon}{c}$

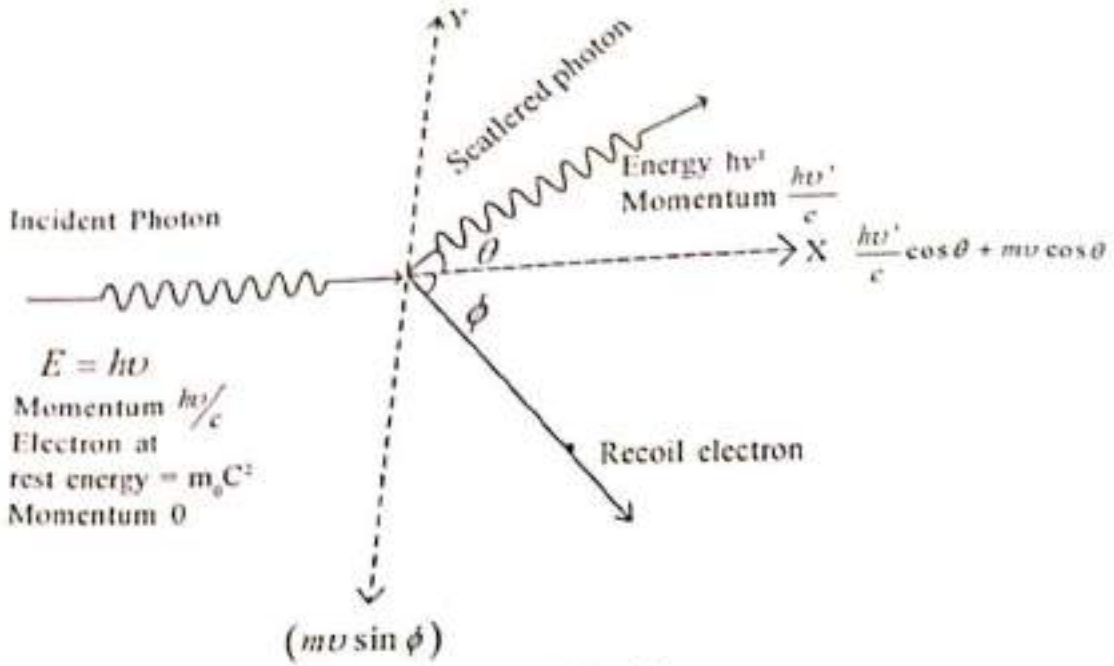
2) The electron are assumed to be free and stationary becouse binding energy of the electron to the atom of order 10 eV, which is very small compared to energy of X-ray photon having order 10eV. The kinetic energy of the electron being of the same order as the binding energy. It is comparatively at rest.

The collesion between high energy photon and the electron is elastic. The energy and the momentum carried by the scattered photon and recoil electron is goverened by the law of conservation of energy and momentum.

Suppose an icident photon with energy and momentum $\frac{h\upsilon}{c}$ strikes an electron at rest. The initial momentum of the electron is zero and the initial energy is only the rest mass energy m_0c^2.

The scattered photon of energy $h\upsilon'$ and momentum $\frac{h\upsilon'}{c}$ moves off in a direction inclined at an angle θ to the original direction.

The electron acquires a momentum $m\upsilon$ and moves at an angle ϕ to the orignial direction. The energy of recoil electron is i.e. shown in below Fig.



[Fig : Compton Effect]

According to principle of conservation of energy

$$h\upsilon + m_0c^2 = h\upsilon' + mc^2 \qquad \text{------ (1)}$$

The principle of conservation along x-direction

$$\frac{h\upsilon}{c} = \frac{h\upsilon'}{c}\cos\theta + m\upsilon\cos\phi$$

multiply by c

$$h\upsilon = h\upsilon'\cos\theta + m\upsilon c\cos\phi \qquad \text{----- (2)}$$

The principle of conservation along y-direction

$$\frac{h\upsilon'}{c}\sin\theta - m\upsilon\sin\phi = 0$$

multiply by c

$$h\upsilon'\sin\theta - m\upsilon c\sin\phi = 0 \qquad \text{-----} (3)$$

From equation (2)

$$m\upsilon c\cos\phi = h\upsilon - h\upsilon'\cos\theta$$

$$m\upsilon c\cos\phi = h(\upsilon - \upsilon'\cos\theta) \qquad \text{-----} (4)$$

from equation (3)

$$m\upsilon c\sin\phi = h\upsilon'\sin\theta \qquad \text{-----} (5)$$

Squaring and adding equation (4) and (5) we get,

$$m^2\upsilon^2c^2\sin^2\phi + m^2\upsilon^2c^2\cos^2\phi = h^2(\upsilon - \upsilon'\cos\theta)^2 + h^2\upsilon'^2\sin^2\theta$$

$$m^2\upsilon^2c^2(\sin^2\phi + \cos^2\phi) = h^2(\upsilon^2 - 2\upsilon\upsilon'\cos\theta + \upsilon'^2\cos^2\theta) + h^2\upsilon'^2\sin^2\theta$$

$$m^2\upsilon^2c^2 = h^2\upsilon^2 - 2\upsilon\upsilon'h^2\cos\theta + h^2\upsilon'^2\cos^2\theta + h^2\upsilon'^2\sin^2\theta$$

$$m^2\upsilon^2c^2 = h^2\upsilon^2 - 2\upsilon\upsilon'h^2\cos\theta + h^2\upsilon'^2(\cos^2\theta + \sin^2\theta)$$

$$m^2\upsilon^2c^2 = h^2(\upsilon^2 - 2\upsilon\upsilon'\cos\theta + \upsilon'^2) \qquad \text{-----} (6)$$

From equation (1) $mc^2 = h(\upsilon - \upsilon') + m_0c^2$ ----- (7)

squaring both side we get

$$m^2c^4 = h^2(\upsilon-\upsilon')^2 + 2h(\upsilon-\upsilon')m_0c^2 + m_0{}^2c^4$$

$$m^2c^4 = h^2(\upsilon^2 - 2\upsilon\upsilon' + \upsilon'^2) + 2h(\upsilon-\upsilon')m_0c^2 + m_0{}^2c^4$$

$$m^2c^4 - m^2\upsilon^2c^2 = \left[h^2(\upsilon^2 - 2\upsilon\upsilon' + \upsilon'^2) + 2h(\upsilon-\upsilon')m_0c^2 + m_0{}^2c^4\right]$$

$$-\left[h^2(\upsilon^2 - 2\upsilon\upsilon'\cos\theta + \upsilon'^2)\right]$$

$$= h^2\upsilon^2 - 2h^2\upsilon\upsilon' + h^2\upsilon'^2 + 2h(\upsilon-\upsilon')m_0c^2 + m_0{}^2c^4$$

$$-h^2\upsilon^2 + 2h^2\upsilon\upsilon'\cos\theta - h^2\upsilon'^2$$

$$m^2c^2(c^2-\upsilon^2) = -2h^2\upsilon\upsilon'(1-\cos\theta) + 2h(\upsilon-\upsilon')m_0c^2 + m_0{}^2c^4 \quad \text{-----} \quad (8)$$

But relativistic mass is given as,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \therefore m^2 = \frac{m_0{}^2}{1-\frac{v^2}{c^2}}$$

$$or \ m^2 = \frac{m_0{}^2}{\frac{c^2-v^2}{c^2}} = \frac{m_0{}^2c^2}{c^2-v^2}$$

Multiply by c^2 both sides we get,

$$m^2c^2 = \frac{m_0{}^2c^4}{c^2 - v^2} \quad \therefore \quad m^2c^2(c^2 - v^2) = m_0{}^2c^4 \qquad \text{----- (9)}$$

equating equation (8) and (9) we get,

$$m_0c^2 = -2h^2\upsilon\upsilon'(1 - \cos\theta) + 2h(\upsilon - \upsilon')m_0c^2 + m_0{}^2c^4$$

$$2h^2\upsilon\upsilon'(1 - \cos\theta) = 2h(\upsilon - \upsilon')m_0c^2$$

$$\frac{(\upsilon - \upsilon')}{\upsilon\upsilon'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

$$\frac{1}{\upsilon'} - \frac{1}{\upsilon} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

Multiply by c we get,

$$\frac{c}{\upsilon'} - \frac{c}{\upsilon} = \frac{hc}{m_0c^2}(1 - \cos\theta)$$

$$But \ \frac{c}{\upsilon'} = \lambda', \ \frac{c}{\upsilon} = \lambda$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta) \quad \text{------ (10)}$$

Thus the change in wavelength is

$$d\lambda = \frac{h}{m_0 c}(1 - \cos\theta) \quad \text{------ (11)}$$

Obviously $d\lambda$ depends upon the angle of scattering only and is independent of the wavelength of the incident radiations as well as nature of the scattering sabstance

$$When\ \theta = 0 \quad Then\ \ d\lambda = \frac{h}{m_0 c}(1 - \cos\theta) = \frac{h}{m_0 c}(1 - 1)$$

$$d\lambda = 0$$

$$\theta = 90 \quad Then\ d\lambda = \frac{h}{m_0 c}(1 - \cos\theta) = \frac{h}{m_0 c}(1 - 0)$$

$$= \frac{6.62 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}}$$

$$d\lambda = 0.024\times10^{-10}\,m = 0.024^{0}A$$

$$\theta = 180 \quad Then\ d\lambda = \frac{h}{m_0c}(1+1) = \frac{h}{m_0c}(2)$$

$$d\lambda = 0.048\times10^{-10}\,m = 0.048^{0}A$$

This is maximum change in wavelength.

Thus the maximum wavelength change possible is $\cong 0.05A^{0}$. The Compton effect cannot give explaination of radiation of wavelength more than $5000\,A^{0}$. For this maximum wavelength change is $0.05A^{0}$ is about 0.001% of the incident wavelength. Hence the Compton effect cannot be observed for visible light rays. From this theory it is assumed that the electron is lossely bounded to the atom that it can be regarded as free but if a photon becomes incident on the electron which is tightly bounded to the atom. The atom as a whole recoil as a result of Compton scattering.

But in case of tightly bound electron, the change in wavelength $d\lambda$ is neglisible for all values of ϕ. This explains the presence of unmodified radiation observed by Compton.

Now divide equation (5) by (4) we get,

$$\frac{m\upsilon c \sin\phi}{m\upsilon c \cos\phi} = \frac{h\upsilon' \sin\theta}{h(\upsilon - \upsilon' \cos\theta)}$$

$$\tan\phi = \frac{\upsilon' \sin\theta}{(\upsilon - \upsilon' \cos\theta)} \qquad \text{-----} \ (12)$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\frac{c}{\upsilon'} - \frac{c}{\upsilon} = \frac{hc}{m_0 c^2}(1 - \cos\theta)$$

$$\frac{1}{\upsilon'} - \frac{1}{\upsilon} = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

$$\frac{1}{\upsilon'} = \frac{1}{\upsilon} + \frac{h}{m_0 c^2}(1 - \cos\theta)$$

$$\frac{1}{\upsilon'} = \frac{1}{\upsilon} + \left(\frac{h}{m_0 c^2}\right).2\sin^2\frac{\theta}{2}$$

$$\frac{1}{\upsilon'} = \frac{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}}{\upsilon}$$

$$\upsilon' = \frac{\upsilon}{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}} \qquad \text{----- (13)}$$

Putting υ' in equation (12) we get

$$\tan\phi = \frac{\frac{\upsilon}{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}}\cdot\sin\theta}{\upsilon - \frac{\upsilon}{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}}\cdot\cos\theta}$$

$$\tan\phi = \frac{\sin\theta}{1+\left(\frac{2h\upsilon}{m_0c^2}\right).\sin^2\frac{\theta}{2}-\cos\theta} = \frac{\sin\theta}{\left[(1-\cos\theta)+\left(\frac{2h\upsilon}{m_0c^2}\right).\sin^2\frac{\theta}{2}\right]}$$

$$\tan\phi = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\left[\sin^2\frac{\theta}{2}+\left(\frac{2h\upsilon}{m_0c^2}\right).\sin^2\frac{\theta}{2}\right]} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\left[2\sin^2\frac{\theta}{2}\left(1+\frac{h\upsilon}{m_0c^2}\right)\right]}$$

$$\tan\phi = \frac{\cos\frac{\theta}{2}}{\left[\sin\frac{\theta}{2}\left(1+\frac{h\upsilon}{m_0c^2}\right)\right]}$$

$$\tan\phi = \frac{\cot\frac{\theta}{2}}{\left(1+\frac{h\upsilon}{m_0c^2}\right)} \qquad \text{----- (14)}$$

This equation gives direction of recoil electron and depends on θ (Recoil angle depends on scattering angle) Also the kinetic energy to recoil electron is equal to energy of incident photon minus energy of scattered photon.

$$K.E. = h\upsilon - h\upsilon'$$

$$K.E. = h\upsilon - h\left(\frac{\upsilon}{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}}\right)$$

$$K.E. = h\upsilon\left(1-\frac{1}{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}}\right)$$

$$K.E. = h\upsilon\left(\frac{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}-1}{1+\left(\frac{h\upsilon}{m_0c^2}\right).2\sin^2\frac{\theta}{2}}\right)$$

$$K.E. = h\upsilon\left(\frac{2.\left(\frac{h\upsilon}{m_0c^2}\right).\sin^2\frac{\theta}{2}}{1+2.\left(\frac{h\upsilon}{m_0c^2}\right).\sin^2\frac{\theta}{2}}\right)$$

This equation shows K. E. of recoiled electron also depends on the scattering angle.

$$K.E. = h\upsilon\left(\frac{2.\sin^2\frac{\theta}{2}}{2.\left(\frac{m_0c^2}{h\upsilon}\right)+\sin^2\frac{\theta}{2}}\right)$$

$$K.E. = h\upsilon\left(\frac{(1-\cos\theta)}{\left(\frac{m_0c^2}{h\upsilon}\right)+(1-\cos\theta)}\right) \qquad \text{----- (15)}$$

:: Solved Problems ::

1) What is the threshold wavelength for a tungsten surface whose work function is 4.5 eV.

Solution : Here, $\phi = 4.5\,\text{eV}$

$= 4.5 \times 1.6 \times 10^{-19}$ Joule.

But $\phi = hv_0$

or $\phi = \frac{hc}{\lambda_0}$

$$\lambda_0 = \frac{hc}{\phi}$$

Here h = 6.6×10^{-34} Joule second.

$$c = 3\times10^8 m/s$$

$$\lambda_0 = \frac{6.6\times10^{-34}\times3\times10^8}{4.5\times1.6\times10^{-19}}$$

$$\lambda_0 = 9.640\times10^{-7} m$$

$$\lambda_0 = 9640\,\overset{0}{\text{A}}\cdot$$

8) Calculate, in electron volt, the energy of a quantum of light of wavelength 5.3×10^{-7} m ·

Solution : Here $\lambda = 5.3 \times 10^{-7}$ m

$$\text{Energy, } E = h\nu = \frac{hc}{\lambda} = \frac{6.624 \times 10^{-34} \times 3 \times 10^{8}}{5.3 \times 10^{-7}} \text{J}$$

$$= 3.75 \times 10^{-19} \text{J} = \frac{3.75 \times 10^{-19}}{1.6 \times 10^{-19}} \text{eV}$$

$$= \mathbf{2.34 \ eV.}$$

:: Multiple Choice Question ::

1) Weins Law holds good in the region of ....
   a) **Shorter wavelength.**
   b) Longer wavelength.
   c) avarage wavalength.
   d) None of these.

2) $\lambda_m \propto \frac{1}{T}$ represents ....
   a) **Weins law.**
   b) Rutherford law.
   c) Plancks law.
   d) None of these.

3) Plancks law reduces to weins law for ....
   a) **Shorter wavelength.**
   b) Longer wavelength.
   c) avarage wavalength.
   d) None of these.

4) For longer wavelength which law holds?
   a) Weins Law.
   b) **Rayleigh-Jeans law.**
   c) Einstein law.
   d) Plancks Law.

5) The absorptive power of a perfectly black body is ....

a) 0.5
b) 1
c) Zero
d) Infinity

6) The spectrum of black body radiation is ....

a) Line.
b) band.
c) continuous.
d) absorption.

7) Plancks radiation law holds for ....

a) Shorter wavelength.
b) Longer wavelength.
c) All wavalengths.
d) None of these.

8) Which of the following phenomenon supports the quantum nature of light ....

a) Interference.
b) Diffraction.
c) Polarisation.
d) Comption effect.

9) The energy of the photon is ....

a) $E = \frac{hc}{\lambda^2}$
b) $E = hc\lambda$
c) $E = \frac{hc}{\lambda}$
d) None of the above

10) The spectral distribution of energy in the black body radiation was investigated by.....

a) Stefan law.
b) Lummer and Pringshim.
c) Rayleigh jeans law.
d) Plancks and Weins law.

11) The value of planks constant is .... J.s

a) 6.62×10^{-34}
b) 6.62×10^{-27}
c) 6.62×10^{-19}
d) None of these

12) Which of the following equation show linear momentum of photon in terms of wavevector

a) $\vec{P} = h\left|\vec{K}\right|$
b) $\vec{P} = \hbar\left|\vec{K}\right|$
c) $P = \frac{h}{2\pi}\left|\vec{K}\right|$
d) $P = \frac{h}{\lambda}$

13) In black body radiation spectrum as increase in temperature (Wavelength at maximum emission) ....

a) Increase
b) Constant
c) Decrease
d) both (a) and (b)

THANK YOU