F.Y.B.SC. PHYSICS

First Semester Paper II

Heat and Thermodynamics

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Chapter 1

Thermal Conductivity

Transference of heat:-

Basically there are three modes of transfer for heat

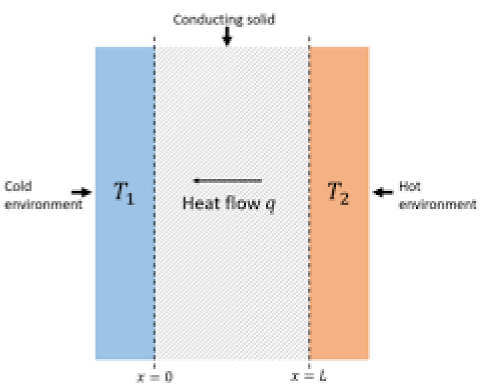
1.Conduction:- The process in which heat is transferred from one point to the other through the substance without the actual motion of the partials is called conduction.

2. Convection:- The process in which heat is transmitted from one place to other by the actual movement of the heated particles is called convection.

3 Radiation:- The process in which heat is transmitted from one place to other directly without the necessity of intervening medium is called radiation.

What is Thermal Conductivity?

Fourier's law of thermal conduction also known as the law of heat conduction is very relevant for heat transfer computation. This principle is applicable for heat transfer between two isothermal planes



The Formula for Thermal Conductivity

Every substance has its own capacity for conducting and transferring the heat. The thermal conductivity of a material is explained by the following formula:

$$K = QX / A\Delta T$$

Also, the above formula can be rearranged to give the value of transfer of heat, as follows:

$$Q = K \times A \; (T_{hot} - T_{cold}) \times t / X$$

The SI unit of this quantity is watts per meter-Kelvin or $Wm^{-1}K^{-1}$. These units will describe the rate of conduction of heat through the material having the unit thickness and for each Kelvin of temperature difference.

K is constant of proportionality, is called coefficient of thermal conductivity of the material.

If A = 1 sqm, [Thot – Tcold] = 1 degree C, t = 1sec. And X = 1cm. Then **Q = K**

M.K.S. unit of coefficient of thermal conductivity is

Kilo cal per meter sec. degree C

Joule / meter sec. degree C

C.G.S. Unit is

cal per Cm sec. degree C

Temperature gradient:- is the rate of change of temperature with respective to the distance is called temperature gradient.

Temperature gradient = T2 - T1 / X

The quantity of temperature gradient is negative, due to temperature decreases as the distance increases from hot end.

$$Q = - KA(d\Theta / dX)\, t$$

Dimension of K is $MLT^{-3}\theta^{-1}$

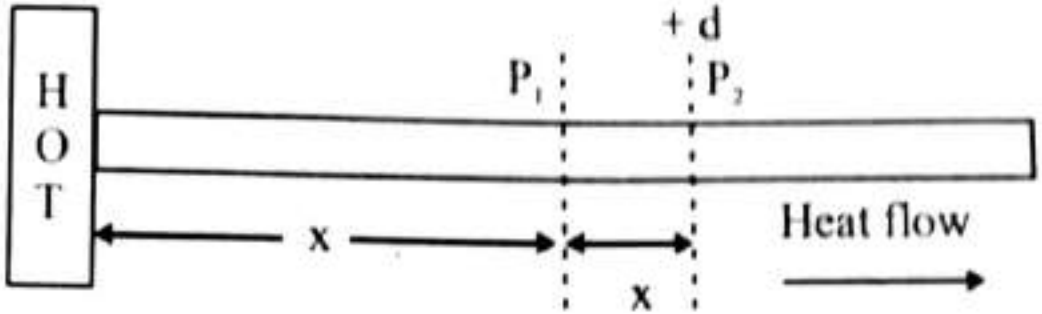
Thermal Resistivity:- It reciprocal of thermal conductivity **(1 / K)** the quantity of heat X / AK is called thermal resistance.

Thermal diffusivity :- the ratio of thermal conductivity per unit volume is called thermal diffusivity.

$$h = K / PS$$

Where **P** is density of the substance

S is the specific heat of the substance



[Fig. 1.2 : Rectilinear flow of heat]

Consider two parallel planes P_1 and P_2 perpendicular to the length of the bar at a distance x and $x + \delta x$ from the hot end. Let θ be the excess of temperature above surroundings of the bar at plane P_1.

The temperature gradient at the plane $P_1 = \frac{d\theta}{dx}$

The excess of temperature at plane $P_2 = \theta + \frac{d\theta}{dx} \cdot \delta x$

and temperature gradient at plane $P_2 = \frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)$

$\therefore$ The quantity of heat flowing through P_1 in one second

$$Q_1 = -KA\frac{d\theta}{dx} \qquad \text{------- (2)}$$

where K-thermal conductivity and A area of cross section.

$\therefore$ Quantity of heat flow through plane P_2 in one second.

$$Q_2 = -KA\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right) \qquad \text{-------}\ (3)$$

Therefore net gain of heat per second by the section δx between plane P_1 and P_2 of the rod.

$$Q = Q_1 - Q_2$$

$$Q = -KA\frac{d\theta}{dx} - \left(-KA\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)\right)$$

$$Q = -KA\frac{d\theta}{dx} + KA\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)$$

$$Q = -KA\frac{d\theta}{dx} + KA\frac{d\theta}{dx} + KA\frac{d^2\theta}{dx^2}\delta x$$

$$\boxed{Q = KA\frac{d^2\theta}{dx^2}\delta x} \quad \text{------- (4)}$$

Case - I : Before the steady state is reached :

The quantity of heat Q is used to raise the temperature of the section and the rest is lost to the surroundings due to radiations before the steady state is reached. Let the rate of rise of temperature of the bar be $\frac{d\theta}{dt}$. The heat used per second to raise the temperature of the rod

= mass of the section × specific heat × raise in temperature per second

$$= (A \times \delta x)\rho \times S \times \frac{d\theta}{dt} \quad \text{------- (5)}$$

where A is the area of cross section of the rod, ρ is the density and S is the specific heat.

The heat lost per second due to radiation.

Q = Emissive power × surface area of the section × temperature excess

$$Q = E \times P \cdot \delta x \times \theta \quad \text{------- (6)}$$

where E is the emissive power of the surface, P is the perimeter, θ is the average excess of temperature of the bar between the planes P_1 and P_2.

$\therefore$ the eqution (5) and (6) becomes

$$Q = A\delta x\rho \times S\frac{d\theta}{dt} + E \times P \cdot dx \cdot \theta$$

Substituting the value of Q from equation 4.

$$KA\frac{d^2\theta}{dx^2}\delta x = A\delta x\rho \times S\frac{d\theta}{dt} + E \times P \cdot dx \cdot \theta$$

$$\boxed{\frac{d^2\theta}{dx^2} = \frac{\rho S}{K}\frac{d\theta}{dt} + \frac{EP}{KA}\theta} \qquad \text{------- (7)}$$

equation (7) is the general equation of the rectilinear flow of heat along a bar.

Case - II : If the heat lost by radiations is negligible :

The bar is completely covered by insulating from surrounding. The heat lost by the radiation emissivity ($E \times P \cdot \delta x \times \theta$) is zero. In that case the total heat gained by the bar is used to raise the temperature of the bar.

The equation (7) become

$$\frac{d^2\theta}{dx^2} = \frac{\rho S}{K}\frac{d\theta}{dt}$$

or
$$\frac{d^2\theta}{dx^2} = \frac{\rho S}{K}\frac{d\theta}{dt} = \frac{1}{h}\frac{d\theta}{dt} \qquad \text{------- (8)}$$

$$\therefore \quad \boxed{\frac{K}{\rho S} = h \text{ is the thermal diffusivity}}$$

Case - III : After the steady state is reached :

At this stage the temperature at every point of the bar become stationary, i.e. $\frac{d\theta}{dt} = 0$

Equation (7) become

$$\frac{d^2\theta}{dx^2} = \frac{EP}{KA}\theta$$

where $\frac{EP}{KA} = \mu^2$

$$\frac{d^2\theta}{dx^2} = \mu^2 \qquad \text{------- (9)}$$

The general solution of this equation is

$$\theta = Ae^{+\mu x} + Be^{-\mu x} \qquad \text{------- (10)}$$

where A and B are two unknown constants to be determined from the boundary conditions.

1) The bar is sufficiently long, steady state condition. The whole of heat energy is lost from the sides as radiation and free end will be at the temperature of the surroundings.

a) If the bar is of infinite length :

The excess of temperature above the surroundings of the hot end be θ_0 and of the other end be zero.

$\therefore \qquad x = 0 \qquad \theta = \theta_0$

from equation 1.10

$$\theta_0 = A + B$$

$$x = \infty, \theta = 0$$

from equation 1.10

$$0 = Ae^{\infty}$$

But e^{∞} cannot be zero

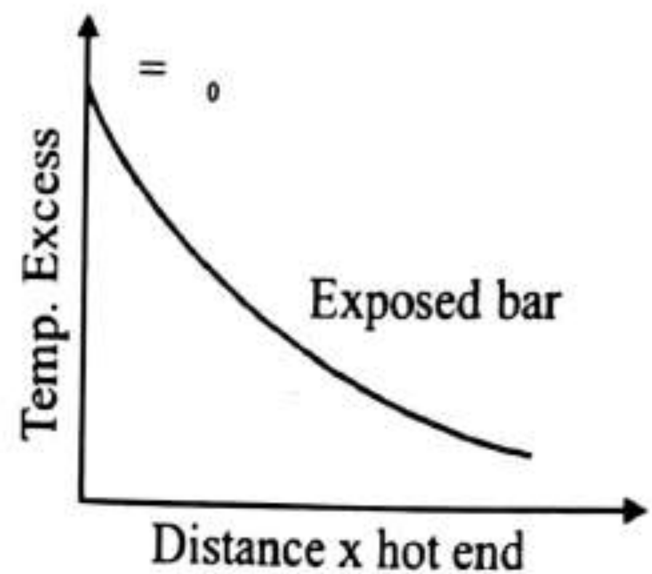
$\therefore A = 0$

$\theta_0 = B$

Substituting the values of A and B in eqution 10 we have

$$\boxed{\theta = \theta_0 e^{-\mu x}} \qquad \text{------- (11)}$$

The equation (11) represents excess of temperature of a point at a distance x from hot end after the steady state is reached.



[Fig. 1.3 : Graphical representation]

b) Suppose the bar is sufficiently long and is of finite length L. Then the boundary conditions are

At $x = 0$

$\theta = \theta_0$

and $\dfrac{d\theta}{dx} = 0$ at $x = L$

The values of A and B in this case are

$$A = \frac{\theta_0}{1 + e^{2\mu L}}$$

$$B = \frac{\theta_0}{1 + e^{-2\mu L}}$$

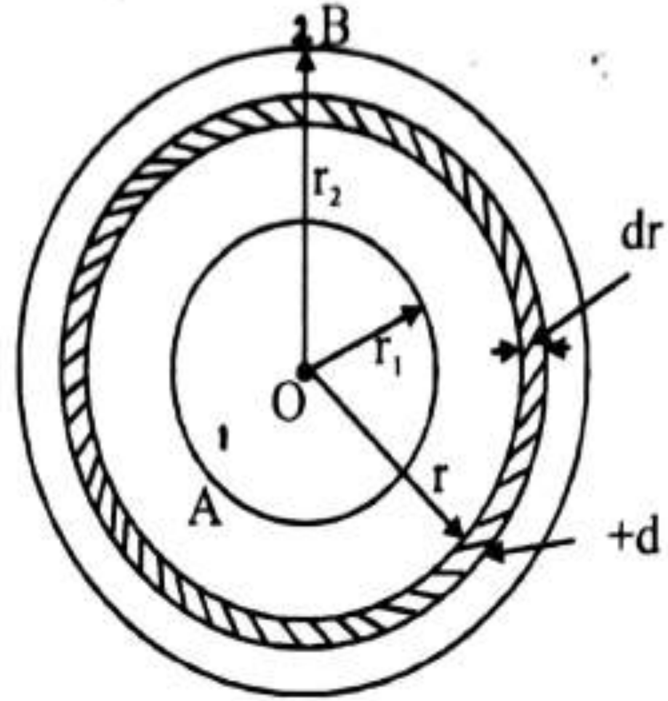
The solution of equation (10) become

$$\boxed{\theta = \theta_0 \left[\frac{e^{\mu x}}{1+e^{2\mu L}} + \frac{e^{-\mu x}}{1+e^{-2\mu L}} \right]} \qquad \text{------- (12)}$$

1.4 Methods of radical flow of heat :

1. Spherical shell method :

Consider two spherical shells A and B of radii r_1 and r_2 respectively. Let a source of heat is electric heating element be placed at the centre O of the shell. The heat is conducted through the shell from inner to outer shell. The temperatures of the inner and outer shells be θ_1 and θ_2 constants after the steady state is reached.



[Fig. 1.3 : Spherical Shell]

Consider an imaginary shell of radius r and thickness dr having temperatures of θ and $\theta + d\theta$ on its inner and the outer surface respectively.

The quantity of heat conducted per second, through this shell,

$$Q = -KA\frac{d\theta}{dr} \qquad \text{-------} (13)$$

$$Q = -K4\pi r^2 \frac{d\theta}{dr} \qquad (\because A = 4\pi r^2)$$

$$dr \qquad 4\pi K$$

$$K = \frac{Q(r_2 - r_1)}{4\pi r_1 r_2 (\theta_1 - \theta_2)} \quad \text{----} \quad (14)$$

from this equation (14) the value of conductivity K can be calculated.

Case : The temperature distribution in the spherical shell may be obtained.

The value of Q from equation (13) and (14), we get,

$$-K \cdot 4\pi r^2 \frac{d\theta}{dt} = \frac{4\pi(\theta_1 - \theta_2) r_1 r_2}{r_2 - r_1}$$

or $$d\theta = -\frac{(\theta_1 - \theta_2) r_1 r_2}{r_2 - r_1} \frac{dr}{r^2}$$

Integrating both sides

$$\int d\theta = -\frac{(\theta_1 - \theta_2) r_1 r_2}{r_2 - r_1} \int \frac{dr}{r^2}$$

$$\theta = -\frac{(\theta_1 - \theta_2) r_1 r_2}{r_2 - r_1} \left(-\frac{1}{r}\right) + C \quad (15)$$

Now in order to obtain the value of constant C, we have,

$$r = r_1,\ \theta = \theta_1$$

$$\therefore \quad \theta_1 = \frac{(\theta_1 - \theta_2) r_1 r_2}{r_2 - r_1} \frac{1}{r_1} + C$$

or
$$C = \theta_1 - (\theta_1 - \theta_2) \frac{r_2}{r_2 - r_1}$$

Substituting it in equation (15) we get

$$\theta = \frac{(\theta_1 - \theta_2) r_1 r_2}{r(r_2 - r_1)} + \theta_1 - (\theta_1 - \theta_2) \frac{r_2}{r_2 - r_1}$$

or
$$\theta = \frac{1}{r_2 - r_1}\left(\frac{(\theta_1 - \theta_2) r_1 r_2}{r} + (r_2 - r_1)\theta_1 - (\theta_1 - \theta_2) r_2\right)$$

$$\theta = \frac{1}{r_2 - r_1}\left(\frac{(\theta_1 - \theta_2) r_1 r_2}{r} + r_2\theta_1 - r_1\theta_1 - \theta_1 r_2 + \theta_2 r_2\right)$$

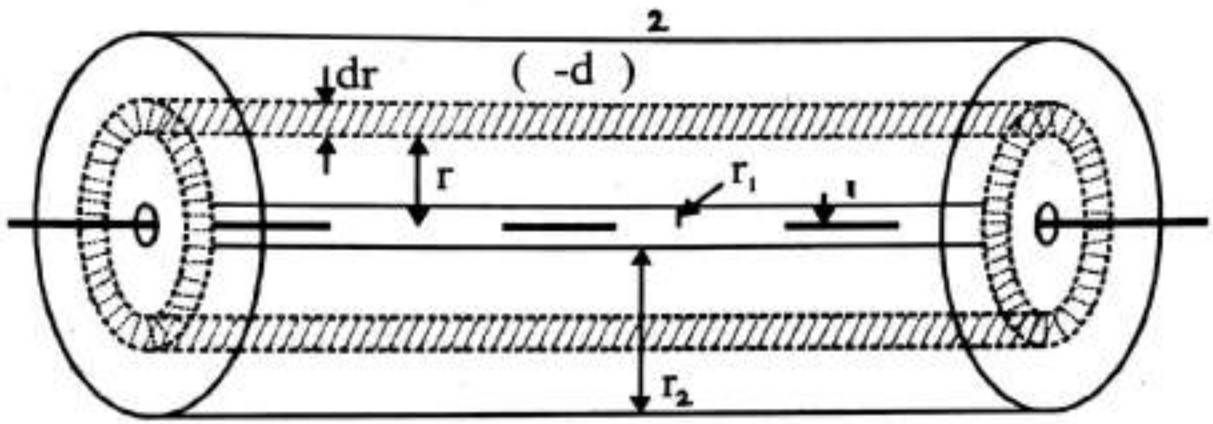
$$\theta=\frac{1}{r_2-r_1}\left(\frac{(\theta_1-\theta_2)r_1r_2}{r}+(\theta_2r_2-r_1\theta_1)\right) \quad \text{------- (16)}$$

The equation (16) gives the temperature of spherical surface of radius r.

2) *Flow of heat along the wall of a cylindrical tube :*

Consider a cylindrical tube of length 'l', inner radius 'r_1' and outer radius 'r_2'. The tube be heated along its axis by placing on electrically heated wire. Heat will conducted radially from inner side towards the outer side across the walls of the tube. When the steady state reached the temperature constant of the inner and outer surface of the tube are θ_1 and θ_2 respectively. $(\theta_1 > \theta_2)$

Consider a cylindrical shell of thickness dr at a distance r from the axis. Let θ and $(\theta-d\theta)$ be the temperatures at r and (r+dr).



[Fig. 1.4 : Cylindrical tube]

The quantity of heat flowing per second across the element

$$Q = -KA\frac{d\theta}{dr} \qquad \text{------- (17)}$$

$$Q = -K2\pi rl\frac{d\theta}{dr} \qquad (\because A = 2\pi rl)$$

$$\therefore\ Q\frac{dr}{r} = -K2\pi l d\theta \qquad \text{------- (18)}$$

or $$Q\left[\log_e \frac{r_2}{r_1}\right] = 2\pi Kl(\theta_1 - \theta_2) \qquad \text{------- (19)}$$

$\therefore$ $$K = \frac{Q\log_e \frac{r_2}{r_1}}{2\pi l(\theta_1 - \theta_2)} \qquad \text{------- (20)}$$

or $$\boxed{K = \frac{Q \times 2.3026 \times \log_{10} \frac{r_2}{r_1}}{2\pi l(\theta_1 - \theta_2)}} \qquad \text{------- (21)}$$

The equation (21) is used to find thermal conductivity of poor conductors given in the form of cylindrical tubes.

Case : To find the temperature θ of the cylindrical shell at a distance r, equating the rate of Q from eqution (17) and (19)

$$Q = -K \cdot 2\pi r l \frac{d\theta}{dr} = \frac{2\pi K l(\theta_1 - \theta_2)}{\log_e \frac{r_2}{r_1}}$$

or $$d\theta = -\frac{(\theta_1 - \theta_2)}{\log_e \frac{r_2}{r_1}} \cdot \frac{dr}{r}$$

Integrating both sides we have,

$$\int d\theta = -\frac{(\theta_1 - \theta_2)}{\log_e \frac{r_2}{r_1}} \cdot \int \frac{dr}{r}$$

$$\theta = -\frac{(\theta_1 - \theta_2)}{\log_e \frac{r_2}{r_1}} \log_e r + C \qquad \text{------- (22)}$$

But when $r = r_1$, $\theta = \theta_1$

$$\theta_1 = \frac{(\theta_1 - \theta_2)}{\log_e \frac{r_2}{r_1}} \log_e r_1 + C$$

or $$C = \theta_1 + \frac{\theta_1 - \theta_2}{\log_e \frac{r_2}{r_1}} \log_e r_1$$

Substituting the value of C in equation (22) we get

$$\theta = -\frac{\theta_1 - \theta_2}{\log_e \frac{r_2}{r_1}} \log_e r + \theta_1 + \frac{\theta_1 - \theta_2}{\log_e \frac{r_2}{r_1}} \log_e r_1$$

$$\theta = \frac{1}{\log_e \frac{r_2}{r_1}} \left[-(\theta_1 - \theta_2) \log_e r + \theta_1 \log_e \frac{r_2}{r_1} + (\theta_1 - \theta_2) \log_e r_1 \right]$$

$$\theta = \frac{1}{\log_e \frac{r_2}{r_1}} \left[-(\theta_1 - \theta_2) \log_e r + \theta_1 \log_e r_2 - \theta_1 \log_e r_1 + \theta_1 \log_e r_1 - \theta_2 \log_e r_1 \right]$$

$$\theta = \frac{1}{\log_e \frac{r_2}{r_1}} \left[-(\theta_1 - \theta_2) \log_e r + \theta_1 \log_e r_2 - \theta_2 \log_e r_1 \right]$$

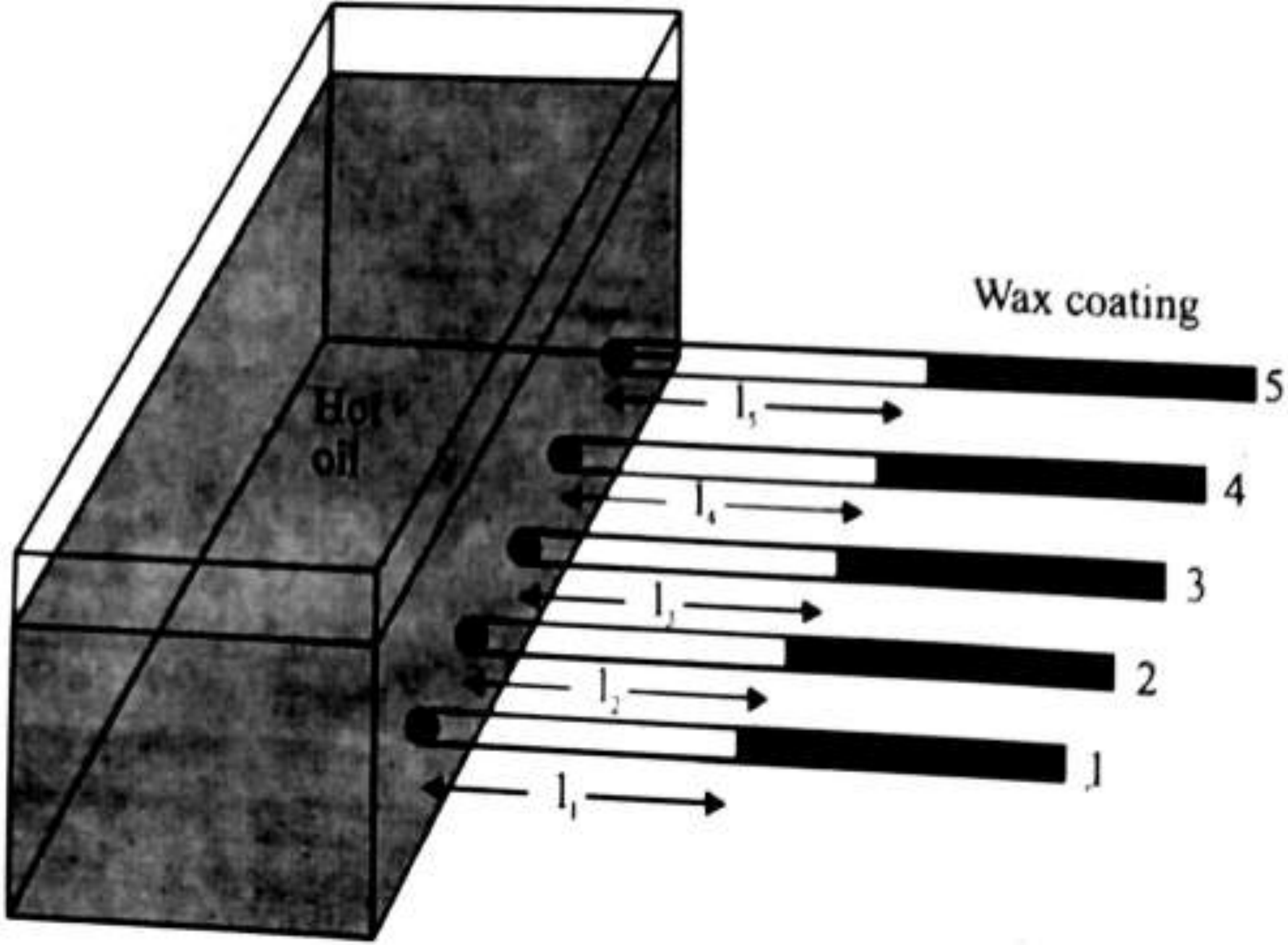
$$\boxed{\theta = \frac{1}{\log_e \frac{r_2}{r_1}} \left[(\theta_1 \log_e r_2 - \theta_2 \log_e r_1) - (\theta_1 - \theta_2) \log_e r \right]} \qquad \text{------- (23)}$$

The equation (23) represents temperature distribution in a cylindrical tube heatedd along the axis in steady state.

1.5 Comparison of conductivities of different metals : Ingen Hausz Experiment :

The experiment is used to compare the thermal conductivities of different materials (Metals). Take a box containing hot water or oil. Materials are taken in the form of long thin rods., identical in size and shape and polished similarly. The emissive power E for all the rods is the same. Rods are fixed at the base of the vessel and their portions outside the vessel are coated uniformaly with wax. Oil or water is maintained at boiling point.

Heat is conducted along the rods and in steady state wax is found to be melted to different lengths depending upon the value of K for different materials.



[Fig. 1.5 : Conductivities of different materials]

Let the lengths up to which the wax has melted be l_1, l_2, l_3... etc. θ_0 be the excess of temperature of heated ends, then the equation representing the temperature distribution along each bar in experiment is

$$\theta = \theta_0 e^{-\mu x}$$

Let θ_m be the excess temperature of the melting point of wax above the room temperature.

As first rod $\qquad \theta_m = \theta_0 e^{-\mu_1 l_1}$

for second rod $\qquad \theta_m = \theta_0 e^{-\mu_2 l_2}$

for third rod $\qquad \theta_m = \theta_0 e^{-\mu_3 l_3}$ and so on

$$\therefore \qquad \theta_m = \theta_0 e^{-\mu_1 l_1} = \theta_0 e^{-\mu_2 l_2} = \theta_0 e^{-\mu_3 l_3} = ...$$

$$\therefore \qquad \mu_1 l_1 = \mu_2 l_2 = \mu_3 l_3 = ...$$

But

$$\mu_1 = \sqrt{\frac{EP}{K_1 A}}$$

$$\mu_2 = \sqrt{\frac{EP}{K_2 A}}$$

$$\mu_3 = \sqrt{\frac{EP}{K_3 A}}$$

Here E, P and A for all the rods are same, as they are identical in size, shape etc.

$$\therefore \qquad l_1\sqrt{\frac{EP}{K_1 A}} = l_2\sqrt{\frac{EP}{K_2 A}} = l_3\sqrt{\frac{EP}{K_3 A}} = ...$$

i.e.
$$\frac{l_1}{\sqrt{K_1}} = \frac{l_2}{\sqrt{K_2}} = \frac{l_3}{\sqrt{K_3}} = ... = \text{Constant}$$

or
$$\boxed{\frac{K_1}{l_1^2} = \frac{K_2}{l_2^2} = \frac{K_3}{l_3^2} = ... = \text{Constant}} \qquad \text{-------} \ (24)$$

i.e. the conductivities of the material of the rod is directly proportional to the square of the length up to which the wax melted on the rod. The equation (24) is used to compare the thermal conductivities of different metals.

1. The rate of flow of heat through a metal bar of area of cross section 1 m^2 when temperature gradient is 1 $^{0}C/m$ under steady state is called,
   a) Thermal resistance
   b) Diffusivity
   c) Thermal conductivity
   d) Resistivity

2. Under steady state, the temperature of a body
   a) Increases with time
   b) decreases with time
   c) does not change with time and is same at all points of the body
   d) does not change with time and can be different at different points of the body.

3. Thermal conduction in metals takes place by
   a) Free electron
   b) bound electron
   c) vibrations of molecules
   d) none of above

4. A single plate of double thickness is made by joining two equal plates of thermal conductivities K_1 and K_2. The equivalent thermal conductivity of the composite plate will be
   a) $\frac{2K_1K_2}{K_1+K_2}$
   b) $\frac{K_1K_2}{K_1+K_2}$
   c) $\frac{K_1}{K_1+K_2}$
   d) $\frac{K_2}{K_1+K_2}$

5. If the density of material is ρ and specific heat is S, then thermal diffusivity is,
   a) $h = \frac{\sigma}{PK}$
   b) $h = \frac{K}{\rho S}$
   c) $h = \frac{K\rho}{S}$
   d) $h = \frac{\rho S}{K}$

6. In the steady state of temperature, the flow of heat across the body depends,
   a) Only upon its thermal conductivity
   b) Upon its thermal conductivity and thermal capacity
   c) Upon its thermal capacity only
   Neither upon thermal conductivity nor upon thermal capacity

7. The SI unit of thermal conductivity is;

a) $Jsm^{-1}/^{0}C$
b) $J/sec.m.^{0}C$
c) $J\ ^{0}C/Sec.m$
d) $J.m/Sec.^{0}C$

8. If l is the length and A area of cross section of a rod and K is termal conductivity of material then the thermal resistance is given by

a) $\frac{Kl}{A}$
b) $\frac{A}{Kl}$
c) $\frac{KA}{l}$
d) $\frac{l}{KA}$

9. The coefficient of thermal conductivity of a metal depends upon,

a) temperature difference between the two sides
b) thickness of the metal plate
c) area of the plate
d) none of above.

10. Four rods with different radii 'r' and length 'l' are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat?

a) r = 1cm, l = 1 m
b) r = 2cm, l = 2m
c) r = 1 cm, l = 0.5 m
d) r = 2cm, l = 0.5m

11. Heat is flowing through two cylindrical rods of same material. The diameter of the rods are in the ratio 1:2 and their lengths in the ratio 2:1. If the temperature difference their ends is the same, then the ratio of amount of heat conducted through them per unit time will be;

a) 1:1
b) 2:1
c) 1:4
d) 1:8

12. A metallic rod is heated at one end continuously. After some time steady state is reached. The flow of heat in the steady state does not depend upon,

a) the area of cross section of the rod
b) the temperature gradient
c) the mass of the rod
d) the time of flow of heat

13. In Ingen-Hauz experiment the thermal conductivity K and length 'l' of the rod up to which wax melt are related as,

a) $\frac{K}{l} = \text{Cons tan t}$

b) $\frac{K^2}{l} = \text{Cons tan t}$

c) $\frac{K}{l^2} = \text{Constant}$

d) $Kl = \text{Cons tan t}$

14. If the density of the material is ρ, specific heat is S, diffusivity is D, then its thermal conductivity K is,

a) $K = \frac{S}{D\rho}$

b) $K = SD\rho$

c) $K = \frac{SD}{\rho}$

d) $K = \frac{\rho S}{D}$

15. The experiment used to compare the thermal conductivities of different metals,

a) Searls method

b) Ingen-Hauz experiment

c) Lee's method

d) None of above

All the students are informed that you should study minimum three slides regurerly.

If any doubt you can contact me or when we will meet we will discuss.

“Happy Dipawali”